Trade Wars and the Reallocation of Market Power in Global Export Markets*

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27 March 2026
Preliminary and Incomplete

Abstract

Trade wars do not just reallocate trade flows—they reshape market structure. Tariffs induce foreign exporters to compress price–cost markups or exit destination markets. We develop a multi-country model with Cournot competition and rich production-network linkages spanning both final goods and intermediate inputs. A central innovation is that firms make endogenous market-participation decisions, allowing entry and exit to interact with variable markups under oligopolistic competition. We provide an analytical characterization of this interaction and quantify its implications in general equilibrium. In a U.S.-centered trade war, endogenous exit by foreign exporters substantially amplifies welfare losses relative to a no-exit benchmark. By contrast, when foreign firms reduce markups to remain active, losses are attenuated: U.S. welfare declines fall from 1.52 to 1.26 percent, and Canada's from 3.0 to 0.4 percent. Production networks further magnify these effects—by roughly a factor of six in our quantitative model.

JEL classification: F12, F13, F41

Keywords: trade war, oligopolistic competition, tariff, markup, welfare, entry, market power

*We thank participants at various seminars and conferences for their comments and suggestions. The views expressed herein are those of the authors and not necessarily those of the Bank of Canada.

1 Introduction

How large are the welfare costs of a trade war? With US tariff rates at their highest levels in nearly a century—following the 2018–2019 escalation and the broader 2025 round—the question is pressing. One issue highlighted by a widely studied class of models, featuring gravity models with monopolistic competition, firm heterogeneity, and variable markups, is that the welfare costs of trade frictions may be moderated by "elusive pro-competitive effects": the welfare losses from higher markups charged by domestic firms are weakly dominated by the reduction in markups charged by foreign firms to consumers. But the overall welfare changes may be driven by more general effects of tariffs on market structure and the level of competition, especially when, as plausible, they cause the exit of large exporters from the market, only partially compensated by the entry of new players.

We reassess the welfare costs of trade wars calling attention to the interactions between two key adjustment margins by firms—markup and entry/exit, unveiling the potential first-order effects of trade wars on market structure and welfare. The Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2018) elusiveness result applies to monopolistically competitive environments in which firms are atomistic and marginal entry has no first-order welfare effect. In oligopolistic markets with granular firm structure, however, active firms are finite and economically non-marginal. Tariffs change the number of active firms in the market, and the exit of one exporter discretely raises the shares of the firms still active, including those from the same origin. This activates a loop: under strategic competition among foreign exporters, larger shares lead continuing firms to raise their markups; higher markups in turn drive profits and shift the entry and exit cutoffs. The loop works also for domestic firms, but in reverse: stronger demand raises profits, keeping more firms active; sales by a larger number of firms compress incumbent markups. In our analysis, the *markup-entry interaction*, denoted by $\mathcal{J}$, plays a central role, as the welfare component missed by summing the pure markup effect (holding firm counts fixed) and the pure entry effect (holding markups fixed). We provide analytical and quantitative support to the notion that this interaction is highly consequential, and key in assessing the lasting effects of trade wars.

The welfare effects of the markup-entry interaction are large because international trade is granular. In Section 2, using firm-level customs data from 11 exporter countries, we document that a typical product-destination is served by just three exporters from a given origin, with a top-two within-origin share of 99.6% (Table 1). At this level of within-origin concentration, each exit reshapes the markup and profit landscape for the remaining sellers. Firm-level evidence from Crowley, Han and Prayer (2024) shows that tariff reductions under trade agreements raise entry, lower markups, and generate opposing within-origin and origin-level share movements—implying the reverse pattern under tariff increases, exactly the margins through which the interaction operates.

Motivated by this evidence, we estimate a multi-country general-equilibrium model with oligopolistic competition in *both* final-demand and intermediate-input markets, endogenous export participation, and production networks calibrated to the World Input-Output Database (WIOD). We discipline the model jointly with aggregate bilateral trade shares and five firm-level tariff elasticities from Crowley, Han and Prayer (2024). The model reproduces a low export-value elasticity, a positive markup elasticity, opposing within-origin and origin-level share movements, and a strong negative entry elasticity. Variable-markup trade models in the Atkeson and Burstein (2008) tradition feature oligopoly in final goods only (Atkeson and Burstein, 2008; Edmond, Midrigan and Xu, 2015) and abstract from production networks. Our two-layer oligopoly structure captures how tariff-induced exit in one market raises input costs in linked markets, amplifying welfare effects through the supply chain.

In a US-centered trade war, three results stand out. First, entry and exit are the dominant welfare margin. The US welfare loss is 0.53 percent in the fixed-entry benchmark, rises to 1.52 percent when entry adjusts with fixed markups, and is 1.26 percent in the full model. When firm counts are held fixed, markup adjustment barely changes aggregate welfare—consistent with the Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2018) elusiveness result. But accounting for entry and exit, markups matter because their adjustment determines which firms stay active: markup-optimization in response to tariff and changing market conditions keeps some exporters from exiting, partially offsetting the variety loss. The markup-entry interaction is even stronger for Canada and Mexico, whose exporters face the sharpest tariff increases and the most concentrated destination markets. Second, the interaction changes both sides of the welfare ledger—preserving variety on the price side while limiting domestic expansion on the income side. Third, production networks amplify these forces by roughly a factor of six: without intermediate-input linkages, the US loss falls from 1.26 to 0.20 percent, because exit in one market raises input costs for linked producers through the supply chain.

To provide analytical insight on the complex mechanisms and channels underlying these results, Section 3 develops a two-country model in the Atkeson and Burstein (2008) tradition that isolates the mechanism with a fixed-entry closed-form benchmark and an exact-hat finite-change system. We show that, under fixed firm counts, markup welfare depends on a cross-origin asymmetry index $\mathcal{A}_0$ that is empirically small (Lemma 1). With endogenous entry, two objects characterize the entry-markup interaction: one is the selection strength ζ_o, which governs how strongly profit changes move active firm counts, and markup sensitivity ξ_o, which governs how strongly per-firm market shares move margins (Proposition 1). To derive a framework to assess the different channels through which trade wars affect welfare, Section 4 develops an exact welfare decomposition that builds on the accounting frameworks of Arkolakis, Costinot and Rodríguez-Clare (2012) and Baqaee and Farhi (2024), extending them to variable markups, ad valorem tariffs, and discrete entry and exit. A four-way counterfactual design—FM×FE, VM×FE, FM×VE, VM×VE—isolates $\mathcal{J}$ as a difference-in-differences.

Related literature. In specifying our model, we draw on Atkeson and Burstein (2008), who develop the workhorse nested-CES oligopoly framework in which markups depend on firm market shares; and Edmond, Midrigan and Xu (2015), who calibrate the framework to Taiwan and find sizable pro-competitive gains—trade disciplines dominant firms and reduces markup distortions—though the overall welfare effect remains close to CES benchmarks. In a follow-up study, Edmond, Midrigan and Xu (2023) decompose the welfare cost of markups into aggregate, misallocation, and entry channels. Relative to these contributions, our model features an additional demand nest and production networks, and our welfare decomposition isolates the markup-entry interaction as a distinct channel. Hsu, Lu and Wu (2020) quantify pro-competitive gains from trade in a head-to-head oligopoly model calibrated to Chinese manufacturing, finding that the constant-markup ACR formula understates gains by 13–31%. Our work also relates to Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2018) showing that, in a broad class of monopolistically competitive gravity models, the welfare effect of variable markups reduces to a single additional sufficient statistic that is empirically small—and elaborate on the "elusiveness" result. De Blas and Russ (2015) study the distribution of markup using heterogeneous-firm Bertrand pricing, and Gaubert and Itskhoki (2021) show how large firms shape trade patterns through granular comparative advantage. On the empirical side, recent work documents tariff pass-through and real-income effects (Amiti, Redding and Weinstein, 2019; Fajgelbaum, Goldberg, Kennedy and Khandelwal, 2020), border-price responses (Cavallo, Gopinath, Neiman and Tang, 2021), buyer-supplier export spillovers (Handley, Kamal and Monarch, 2023, 2025), supply-chain rerouting (Freund, Mattoo, Mulabdic and Ruta, 2023; Alfaro and Chor, 2025), and tariff-induced disruption of firm-to-firm supply relationships (Grossman, Helpman and Redding, 2024). In oligopolistic markets with granular firm structure, the markup-entry interaction generates welfare effects an order of magnitude larger than the pure markup channel, even at the Atkeson and Burstein (2008) benchmark where the within- and across-origin substitution elasticities coincide. As Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2018) note, their analysis may not extend to oligopoly, where aggregation across finite firms invalidates the Pareto-based sufficient statistics—a point echoed by Antràs (2025), who identifies the welfare implications of oligopolistic trade as a major open question for the field.

On the welfare-accounting side, our decomposition builds on and bridges two influential approaches. Arkolakis, Costinot and Rodríguez-Clare (2012) derive an exact welfare formula for a broad class of trade models, expressing welfare changes in terms of domestic expenditure shares and the trade elasticity; we extend their accounting framework to variable markups and ad valorem tariffs. Melitz and Redding (2015) show that the ACR sufficient-statistics result is a knife-edge case: away from Pareto productivity, endogenous firm selection creates a new welfare margin that cannot be captured by aggregate trade shares alone. Our paper pushes further—under oligopoly, entry and exit change not only average productivity but also incumbent markups, generating the non-additive interaction we quantify. Bai, Jin and Lu (2024) show that firm-level distortions can cause reallocation to offset gains from trade, another departure from ACR that operates through the extensive margin. Baqaee and Farhi (2024) decompose welfare into technology and allocative-efficiency components through production networks, but for a fixed set of producers; we build on their framework to accommodate discrete entry and exit. Our seven-channel decomposition nests both approaches and connects them through a Sato-Vartia exact price-index representation incorporating the Feenstra (1994) variety margin. Feenstra and Weinstein (2017) develop a structural translog model with endogenous markups and entry/exit to quantify both markup and variety effects on US welfare; we embed their logic augmented with variety changes in a general-equilibrium accounting framework with production networks.

On the quantitative modeling side, the standard monopolistically competitive gravity framework combines CES preferences, Pareto productivity, and free entry. In that environment, marginal entrants are welfare-neutral beyond variety. Under oligopoly, by contrast, entry also changes incumbent markups, creating the non-additive interaction we quantify. To our knowledge, our model is the first to feature two-layer oligopoly—Cournot competition in both final-demand and intermediate-input markets—embedded in a full production-network structure. Most quantitative trade models feature competitive pricing—possibly with IO linkages (Eaton and Kortum, 2002; Caliendo and Parro, 2015)—oligopoly without production networks (Edmond, Midrigan and Xu, 2015), supply-chain frictions without strategic pricing (Grossman, Helpman and Redding, 2024), or tariff-escalation motives through scale economies in vertical chains (Antràs, Fort, Gutiérrez and Tintelnot, 2024). Combining oligopoly with production networks, our model captures how market-power effects propagate along supply chains—a channel that proves quantitatively dominant.

Roadmap. Section 2 documents the granularity of export markets using firm-level customs data. Section 3 isolates the markup-entry interaction in a two-country model with a fixed-entry closed-form benchmark and an exact-hat finite-change system. Section 4 develops the exact welfare decomposition and the four-way counterfactual design for the multi-country model. Section 5 presents the quantitative model with two-layer oligopoly and production networks, and describes the joint calibration. Section 6 quantifies the welfare effects of a US-centered trade war. Section 7 concludes. The Online Appendix contains proofs, data construction, and implementation details.

2 The Granularity of Export Markets

In our model, firms compete at three levels: within an origin group (among firms from the same exporting country), across origins (between countries' exporters in a destination), and across industries. Within-origin granularity drives the markup-entry interaction: when only a handful of firms from a given origin sell a product in a destination, the exit of one exporter discretely raises the shares and markups of the others.

Data. We use the universe of firm-level customs records from 11 exporter countries—Albania, Bulgaria, Burkina Faso, China, Egypt, Malawi, Mexico, Peru, Senegal, Uruguay, and Yemen—covering exports to 165 destinations over 2000–2013. We measure concentration at the origin–product–destination–year level: for each exporting origin, product, and destination, we count the number of active firms and compute their sales distribution. After harmonizing product codes across HS revisions into 3,646 time-consistent products, the dataset contains 13.3 million firm–product–origin–destination–year observations. Online Appendix A details the data construction.

Concentration. Table 1 reports within-origin concentration. Pooling all destinations (Panel a), the median origin–product–destination cell has just three exporters, a Herfindahl-Hirschman index of 0.65, and a top-two firm share of 99.6%. Even in the large and diversified US market (Panel b), within-origin concentration remains high: the median number of exporters from a given origin is eight, the median top-two share is 88.2%, and the median HHI is 0.50.

Entrant size. Conditional on origin–product–destination cells with at least one incumbent and one entrant, the median entrant captures 37% of within-origin sales in the pooled sample and 16% in the US. The entry or exit of a single firm therefore substantially reshapes within-origin concentration and the competitive environment facing remaining sellers from that origin.

The concentration patterns are robust across destinations and industries (Online Appendix Tables A-3–A-5). They establish that the competitive arena relevant for pricing—an origin's exporters competing within a product-destination market—is typically small, exactly the environment in which markup adjustments and entry decisions interact.

3 A Simplified Model with Markup-Entry Interactions

In this section, we rely on a simplified version of the Atkeson–Burstein model to provide economic insights on the drivers and welfare implications of the interaction between markups and entry. Online Appendix Sections B.1.7 and B.1.8 report the full derivations, the exact-hat AB system used

Table 1: Within-origin export concentration by product-destination

	25th Percentile	Median	75th Percentile
(a) All destination markets			
(i) Number of firms	1.00	3.00	7.00
(ii) Herfindahl-Hirschman Index	0.35	0.65	1.00
(iii) Top-2 market share	75.0%	99.6%	100%
(iv) Cumulative market share cond. on ≥ 1 incumbent and ≥ 1 entrant			
– Incumbents	31.2%	63.1%	86.7%
– Entrants	13.3%	36.9%	68.8%
(b) US market			
(i) Number of firms	2.00	7.00	24.00
(ii) Herfindahl-Hirschman Index	0.25	0.50	0.92
(iii) Top-2 market share	61.6%	89.2%	100%
(iv) Cumulative market share cond. on ≥ 1 incumbent and ≥ 1 entrant			
– Incumbents	49.4%	81.9%	95.2%
– Entrants	4.8%	18.1%	51.6%

Note: Concentration is measured at the consolidated-HS6-product–origin–destination–year level, using an unbalanced panel of the universe of firms exporting from 11 origins to 165 destinations. Panel (a) pools all destinations; Panel (b) restricts to the United States. Panel (iv) reports cumulative incumbent and entrant within-origin shares conditional on cells with at least one incumbent and one entrant, where incumbents sell in both t and $t-1$ and entrants sell in t but not in $t-1$.

below, the finite-K heterogeneous-firm extension, and the more general nested-demand extension with $\sigma > \rho$.

3.1 A Two-country Economy with Fixed-cost Heterogeneity and Entry.

Consider two countries, Home and Foreign, each populated by a unit mass of consumers. The two countries share the same primitives apart from bilateral trade frictions. There are J symmetric sectors. Preferences follow a two-nest CES structure: varieties within a sector are aggregated with elasticity $\rho > 1$, and sectors are aggregated with elasticity $\eta > 1$, where $\rho > \eta$.

In a pre-tariff benchmark, firms are symmetric within each origin. Each sector in a given destination market contains n_D domestic sellers and n_F foreign exporters. All active firms within an origin share a common productivity φ and produce with a linear technology $y = \varphi \cdot l$, where l is labor. Labor is the numeraire ($w = 1$) in both countries.

We abstract from iceberg trade costs and write $\tau \geq 1$ for the gross tariff wedge. The marginal cost of a domestic seller is $1/\varphi$; the tariff-inclusive marginal cost of a foreign exporter is τ/φ.[1]

To capture endogenous entry and exit without losing tractability, we let potential firms within an origin $o \in \{D, F\}$ share the same marginal cost τ_o/φ, with $\tau_D = 1$ and $\tau_F = \tau$, but face different operating costs drawn from a distribution $G_o(f)$ with mass M_o of potential sellers. Entry and exit are then generated by the cutoff rule $f \leq \pi_o^{\text{var}}$, where operating profits are

$$\pi_D = \left(1 - \frac{1}{\mu_D}\right) \frac{S_D E}{n_D}, \qquad \pi_F = \left(1 - \frac{1}{\mu_F}\right) \frac{(1 - S_D)E}{\tau\, n_F}, \tag{1}$$

where μ_o is the markup of firms from origin o, S_o is origin o's market share in the destination market; n_o is the mass of active firms; and E is an aggregate demand shifter; π_o is gross operating profit before the fixed cost is paid. It is also the cutoff object: the marginal active firm in origin o satisfies $f_o^* = \pi_o^{\text{var}}$ and therefore earns zero net profit. The mass of active firms satisfy the exact cutoff equations

$$n_D = M_D G_D(\pi_D)\,, \qquad n_F = M_F G_F(\pi_F)\,. \tag{2}$$

The economic role of the fixed-cost distribution is straightforward: it determines how much firm mass sits close to the zero-profit cutoff. If many firms are close to breakeven, a small profit change triggers a large entry or exit response. If few firms are near the cutoff, the same profit change mostly shows up as larger or smaller surviving firms. Thus fixed-cost heterogeneity is enough to generate endogenous firm counts while preserving the symmetric-within-origin state variable

[1] The Online Appendix retains the more general notation when useful, but all Section 2 formulas are stated in terms of the tariff wedge alone.

$H_o = 1/n_o$.

For analytical tractability, we use a power-CDF family for G_o. The power-CDF implies a constant elasticity of active firm counts with respect to the profit cutoff, so the whole fixed-cost block can be summarized by one selection parameter. That gives endogenous entry and exit, keeps the pricing and welfare system closed form.[2] Concretely, if

$$G_o(f) = \left(\frac{f}{\bar{f}_o}\right)^{a_o}, \qquad a_o > 0,$$

for $0 \le f \le \bar{f}_o$, then

$$n_o = M_o \left(\frac{\pi_o^{\text{var}}}{\bar{f}_o}\right)^{a_o}.$$

Because variable profits are proportional to $z_o S_o E / n_o$, this implies $n_o^{1+a_o} \propto (z_o S_o E \tau_o^{-1})^{a_o}$ and therefore

$$\hat{n}_o = \left(\hat{z}_o \hat{S}_o \hat{E} \hat{\tau}_o^{-1}\right)^{\zeta_o}, \qquad \zeta_o = \frac{a_o}{1 + a_o} \in (0, 1).$$

So the power parameter a_o governs how sharply mass accumulates near the cutoff, while ζ_o is the exact elasticity of active firms with respect to the profit shifter.

3.2 The Welfare Relevance of Markup-Entry Interactions

In the symmetric benchmark, equilibrium prices take the form $p_D = \mu_D/\varphi$ for domestic sellers and $p_F = \mu_F \tau/\varphi$ for foreign exporters, where μ_D and μ_F are endogenous gross markups. The equilibrium share condition satisfies

$$\frac{S_D}{1 - S_D} = \left(\frac{n_D}{n_F}\right) \cdot \left(\frac{\mu_D(S_D)}{\mu_F(S_D) \cdot \tau}\right)^{1-\rho}, \tag{3}$$

which is nonlinear because markups depend on firm market share S_o/n_o, and both S_o and n_o respond to the tariff. In the AB environment, entry matters because it changes the market share of each surviving firm.

The sectoral price index takes the compact form

$$P_s = \frac{\mu_D}{\varphi} \cdot n_D^{\frac{1}{1-\rho}} \cdot S_D^{\frac{1}{\rho-1}}. \tag{4}$$

With J symmetric sectors, the aggregate price index is $P = J^{1/(1-\eta)} \cdot P_s$. In general equilibrium

[2] In Online Appendix B.1.8, we introduce an extension where we allow for heterogeneity in the productivity of the firms.

accounting with tariff revenue rebated to consumers, total expenditure satisfies

$$E = \frac{L\tau}{\tau\frac{S_D}{\mu_D} + \frac{1-S_D}{\mu_F}}, \tag{5}$$

so welfare $W = E/P$ admits the closed form

$$W = \frac{L\tau\varphi}{J^{1/(1-\eta)}\,\mu_D\, n_D^{\frac{1}{1-\rho}}\, S_D^{\frac{1}{\rho-1}}\left(\tau\frac{S_D}{\mu_D} + \frac{1-S_D}{\mu_F}\right)}. \tag{6}$$

We compare four counterfactual cases to isolate the interaction between markup adjustment and entry/exit on welfare:

$$\text{Interaction } \mathcal{J} = \underbrace{(\Delta\log W^{(4)} - \Delta\log W^{(1)})}_{\text{Total}} - \underbrace{(\Delta\log W^{(2)} - \Delta\log W^{(1)})}_{\text{Pure markup}} - \underbrace{(\Delta\log W^{(3)} - \Delta\log W^{(1)})}_{\text{Pure entry}}. \tag{7}$$

where case 1 holds both markups and the set of active firms fixed at their pre-tariff values; Case 2 holds the set of active firms fixed but lets markups adjust; Case 3 allows entry and exit but holds markups fixed; Case 4 is the full equilibrium in which both margins adjust jointly.

If either markups are constant or active firm counts are fixed, this interaction is zero. In the AB model it is generally nonzero because entry and exit change firm market shares, which changes markups, which then feed back into relative prices, origin shares, and entry incentives. Because exporter participation adjusts discretely when there are only a few firms per market, this non-additivity need not be small even for moderate tariff changes.[3]

We start with Lemma 1 illustrating an analogue of the Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2018) benchmark inside our oligopolistic environment.

Lemma 1 (Fixed-entry markup benchmark)**.** *Let $x \equiv \log(\tau_1/\tau_0)$ denote the tariff-induced foreign cost wedge and let $\beta_{\lambda,o} \equiv \partial\log\mu_o/\partial\log\lambda_o|_0$ be the fixed-entry elasticity of markups with respect to the origin share λ_o.*[4] *In the two-country AB benchmark with within-origin symmetry,*

$$\beta_{\lambda,o} = \mu_{o,0}H_{o,0}S_{o,0}\left(\frac{1}{\eta} - \frac{1}{\rho}\right), \qquad H_{o,0} \equiv \frac{1}{n_{o,0}}. \tag{8}$$

Holding firm counts fixed and linearizing the two-origin CES block around the pre-tariff equilibrium then yields

$$\mathcal{M}^P \approx -\Psi_0\,\mathcal{A}_0\,x, \tag{9}$$

[3] Online Appendix B.1.7 gives the derivation of the exact-hat AB system used here and the richer nested-demand extension with $\sigma > \rho$.

[4] Here $\lambda_D \equiv S_D$ and $\lambda_F \equiv 1 - S_D$.

where

$$\Psi_0 \equiv \frac{(\rho-1)S_{D,0}(1-S_{D,0})\left(\frac{1}{\eta}-\frac{1}{\rho}\right)}{D^{10}}, \qquad \mathcal{A}_0 \equiv \mu_{D,0}H_{D,0}S_{D,0} - \mu_{F,0}H_{F,0}(1-S_{D,0}), \tag{10}$$

and

$$D^{10} = 1 + (\rho-1)S_{D,0}(1-S_{D,0})\left(\frac{1}{\eta}-\frac{1}{\rho}\right)\left(\mu_{D,0}H_{D,0} + \mu_{F,0}H_{F,0}\right). \tag{11}$$

Thus first-order fixed-entry markup welfare depends only on the asymmetry index $\mathcal{A}_0$. If $\mathcal{A}_0 = 0$, then $\mathcal{M}^P = 0$ to first order.

When active firm counts are fixed, tariffs still raise the markup of the expanding origin and lower the markup of the contracting origin. But those two forces mostly offset in welfare because the sector price index is a share-weighted average of origin prices, and the two origins move in opposite directions. What survives is only the residual asymmetry index $\mathcal{A}_0$, which combines baseline import exposure, firm concentration, and markups. That is the precise sense in which fixed-entry markup welfare is elusive.

In the Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2018) baseline, the Pareto/free-entry structure makes the mass of entrants invariant to trade costs, and the marginal varieties that enter or exit at the cutoff are first-order welfare-neutral because they are priced at the choke price. Entry therefore does not create a new welfare margin, so the fixed-entry benchmark is also the full equilibrium. In our setting, by contrast, the fixed-entry benchmark is only a comparison object: once active firm counts respond, entry is no longer neutral because each survivor carries positive market share.

The following Corollary highlights a knife-edge result.

Corollary 1 (Symmetry is a knife-edge benchmark)**.** *If $\mu_{D,0} = \mu_{F,0}$, $H_{D,0} = H_{F,0}$, and $S_{D,0} = 1/2$, then $\mathcal{A}_0 = 0$ and therefore $\mathcal{M}^P = 0$ to first order.*

With fixed entry, exact symmetry kills the first-order fixed-entry welfare effect by construction. Away from exact symmetry, fixed-entry markup welfare is generally nonzero, but it is still disciplined by the single statistic $\mathcal{A}_0$. Once entry is endogenous, however, symmetry is no longer sufficient to kill the mechanism itself: a tariff changes the number of active firms, which changes per-firm market share, which changes markups and profits.

To see what changes once entry is endogenous, recall that the perceived demand elasticity is

$$\varepsilon_o^{AB} = \left[\frac{1}{\rho} + \left(\frac{1}{\eta}-\frac{1}{\rho}\right)s_o\right]^{-1} \equiv (z_o)^{-1}, \tag{12}$$

so markups depend only on firm market share $s_o \equiv S_o/n_o$. Entry still matters because tariffs change the number of active firms, and therefore change the market share of each surviving firm even when within-origin concentration has no separate role.

Under the power-CDF fixed-cost specification, the exact cutoff equations imply

$$\hat{n}_D = \left(\hat{z}_D \hat{S}_D \hat{E}\right)^{\zeta_D}, \qquad \hat{n}_F = \left(\hat{z}_F \hat{S}_F \hat{E} \hat{\tau}^{-1}\right)^{\zeta_F}, \tag{13}$$

where hats denote gross changes relative to the pre-tariff equilibrium, $\hat{S}_D \equiv S_D/S_{D,0}$, $\hat{S}_F \equiv (1 - S_D)/(1 - S_{D,0})$, $\hat{\tau} \equiv \tau_1/\tau_0$, and $\zeta_o \in (0, 1)$ is the exact selection coefficient implied by the fixed-cost distribution. Higher profit margins keep more firms active, and foreign tariffs push directly against foreign entry through the factor $\hat{\tau}^{-1}$.

Recall that the parameter ζ_o is the key selection parameter. It measures how strongly an origin-level profit shock changes the number of active firms. If ζ_o is near zero, the entry margin is weak: tariffs mostly change the scale of surviving firms, not the number of firms serving the market. If ζ_o is large, many firms sit close to the cutoff, so small profit changes induce large entry and exit responses.

Proposition 1 below shows exactly how markup adjustment changes entry.

Proposition 1 (AB benchmark: exact markup-entry loop)**.** *Suppose $\sigma = \rho$. Define the baseline markup-sensitivity coefficient*

$$\xi_o \equiv \frac{\left(\frac{1}{\eta} - \frac{1}{\rho}\right) s_{o,0}}{\frac{1}{\rho} + \left(\frac{1}{\eta} - \frac{1}{\rho}\right) s_{o,0}} \in (0, 1) \tag{14}$$

and the baseline markup coefficient

$$\kappa_{o,0} \equiv \mu_{o,0} \left(\frac{1}{\eta} - \frac{1}{\rho}\right) s_{o,0}. \tag{15}$$

Then the exact hat system satisfies

$$\hat{z}_o = 1 + \xi_o(\hat{s}_o - 1), \qquad \hat{\mu}_o = \left[1 + \kappa_{o,0}(1 - \hat{s}_o)\right]^{-1}, \qquad \hat{s}_o \equiv \frac{\hat{S}_o}{\hat{n}_o}, \tag{16}$$

and eliminating $\hat{n}_o$ from (13) yields one exact scalar loop per origin:

$$\hat{s}_o = \hat{S}_o^{1-\zeta_o} \left(\hat{E} \hat{\tau}_o^{-1} \left[1 + \xi_o(\hat{s}_o - 1)\right]\right)^{-\zeta_o}, \qquad \hat{\tau}_D = 1, \qquad \hat{\tau}_F = \hat{\tau}. \tag{17}$$

A higher margin raises the profit-margin object z_o, which raises variable profits and keeps more firms active through the cutoff equation. With more active firms as demand spreads across additional active firms, per-firm market share s_o falls, which feeds back into markups. The coefficient

ζ_o controls how much of an origin-level demand shock shows up as entry rather than firm growth; ξ_o controls how strongly per-firm share moves margins. The loop is strong only when both are large. The foreign side need not mirror the domestic side because the foreign cutoff equation also contains the direct tariff wedge $\hat{\tau}^{-1}$.

Proposition 2 reduces the exact equilibrium to three unknowns: domestic per-firm share, foreign per-firm share, and total expenditure. Equation (18) is the relative-price condition.

Proposition 2 (Determinants of the welfare interaction)**.** *Define the baseline expenditure weights*

$$\omega_{D,0} \equiv \frac{\tau_0 S_{D,0}/\mu_{D,0}}{\tau_0 S_{D,0}/\mu_{D,0} + (1 - S_{D,0})/\mu_{F,0}}, \qquad \omega_{F,0} \equiv 1 - \omega_{D,0},$$

and let $(\hat{s}_D^r, \hat{s}_F^r, \hat{E}^r)$ *solve*

$$1 = \frac{\hat{s}_F^r}{\hat{s}_D^r} \left(\frac{\hat{\mu}_D^r}{\hat{\mu}_F^r \hat{\tau}} \right)^{1-\rho}, \tag{18}$$

$$1 = S_{D,0} \hat{S}_D^r + (1 - S_{D,0}) \hat{S}_F^r, \tag{19}$$

$$\hat{E}^r = \frac{\hat{\tau}}{\omega_{D,0} \hat{\tau} \, \hat{S}_D^r / \hat{\mu}_D^r + \omega_{F,0} \hat{S}_F^r / \hat{\mu}_F^r}, \tag{20}$$

where

$$\hat{z}_o^r = 1 + \xi_o(\hat{s}_o^r - 1), \qquad \hat{\mu}_o^r = [1 + \kappa_{o,0}(1 - \hat{s}_o^r)]^{-1}, \qquad \hat{S}_o^r = (\hat{s}_o^r)^{1+\zeta_o} \left(\hat{z}_o^r \hat{E}^r \hat{\tau}_o^{-1} \right)^{\zeta_o}, \tag{21}$$

with $\hat{\tau}_D = 1$ *and* $\hat{\tau}_F = \hat{\tau}$. *Apply the same reduced system under the regime restrictions*

(00): $\hat{\mu}_o^r = \hat{z}_o^r = 1, \quad \hat{n}_o^r = 1, \quad \hat{s}_o^r = \hat{S}_o^r,$

(10): $\hat{n}_o^r = 1, \quad \hat{s}_o^r = \hat{S}_o^r,$

(01): $\hat{\mu}_o^r = \hat{z}_o^r = 1,$

(11): both margins free.

Then whenever entry is free, active firms are recovered from $\hat{n}_o^r = \hat{S}_o^r / \hat{s}_o^r$*; otherwise* $\hat{n}_o^r = 1$. *Regime-r welfare is*

$$\hat{W}^r = \hat{E}^r \left(\hat{\mu}_D^r \right)^{-1} \left(\hat{s}_D^r \right)^{-1/(\rho-1)}, \tag{22}$$

and the welfare interaction is

$$\mathcal{J} = \log\left(\frac{\hat{W}^{11} \hat{W}^{00}}{\hat{W}^{10} \hat{W}^{01}} \right). \tag{23}$$

Note that per equation (18), domestic firms gain market share when their effective consumer price falls relative to foreign exporters. Per equation (21), a rise in $\hat{s}_o$ raises margins through $\hat{z}_o$ and $\hat{\mu}_o$. It also raises origin expenditure share and active firms, with the strength of that response governed by ζ_o. Per equation (20) total expenditure is high when the expenditure-weighted effective cost of the consumption basket is low. Once $(\hat{s}_D, \hat{s}_F, \hat{E})$ are known, S, n, and μ follow exactly.

The proposition highlights that five objects govern $\mathcal{J}$: baseline import exposure $(S_{D,0}, 1-S_{D,0})$, selection strength (ζ_D, ζ_F), markup sensitivity (ξ_D, ξ_F), the foreign tariff wedge $\hat{\tau}$, and the GE expenditure weights $(\omega_{D,0}, \omega_{F,0})$. Import exposure determines how much demand is available to be reallocated across origins in the first place. Selection strength determines how much of that reallocation shows up as entry and exit rather than simply larger surviving firms. Markup sensitivity determines how strongly those firm-level share changes move margins. The tariff wedge matters because it hits foreign profits directly and therefore makes foreign exit especially responsive. The GE weights matter because they determine how much the domestic relief and the foreign deterioration feed into aggregate expenditure. If either selection or markup sensitivity is weak, the markup-entry loop is weak. The interaction becomes large only when both are strong.

3.3 A Numerical Illustration

Figure 1 quantifies the markup-entry interaction effect in response to a tariff change from 5% to 15%. It plots the exact interaction as a percent of the *baseline full-model welfare change*, defined as $|\Delta \log W^{11}_{\text{base}}|$ for the 5% → 15% benchmark under the baseline calibration. Panel A varies entry elasticity, measured by ζ, nesting the exact fixed-entry benchmark at $\zeta = 0$. Panel B varies market power, measured directly by baseline firm market share $s_0 = S_0/n_0$; the hollow marker at $s_0 = 0$ indicates the competitive limit.

Along the entry-elasticity sweep, the interaction is zero at the fixed-entry limit, positive only when entry responds weakly to profits, and turns negative around $\zeta = 0.49$. The reason is that with a higher entry elasticity, markup-induced profit changes move more firms across cutoffs. At the same time, a higher market power makes entry/exit move surviving firms' markups more strongly. Once firms are already large, further increases in s_0 still keep the interaction negative, but the incremental effect becomes weaker over the plotted range. Along the market-power sweep, the interaction is zero in the no-market-power limit, negative throughout the plotted interior, reaching its largest magnitude around $s_0 \approx 0.16$.

At the baseline calibration (marked by the virtual line), the interaction is about -127% of the baseline full-model welfare change. Once the economy already starts from a positive tariff, allowing markups to adjust mainly works by preserving foreign firms that would otherwise exit, and therefore by limiting the domestic reallocation that makes pure entry relatively favorable in this interval.

In conclusion, the exact-hat representation above is not a local approximation. It matches the exact nonlinear four-case solver to within 10^{-12} numerical tolerance over a 50-point tariff grid from 1% to 50%: the maximum absolute error in $\mathcal{J}$ is below 10^{-12}, and the maximum welfare error in any case is below 10^{-12}. Figure 1 therefore plots exact nonlinear interaction effects. Around the symmetric free-trade benchmark, local derivative formulas are still useful for intuition, but the exact-hat system is the right object for quantifying finite tariff changes.

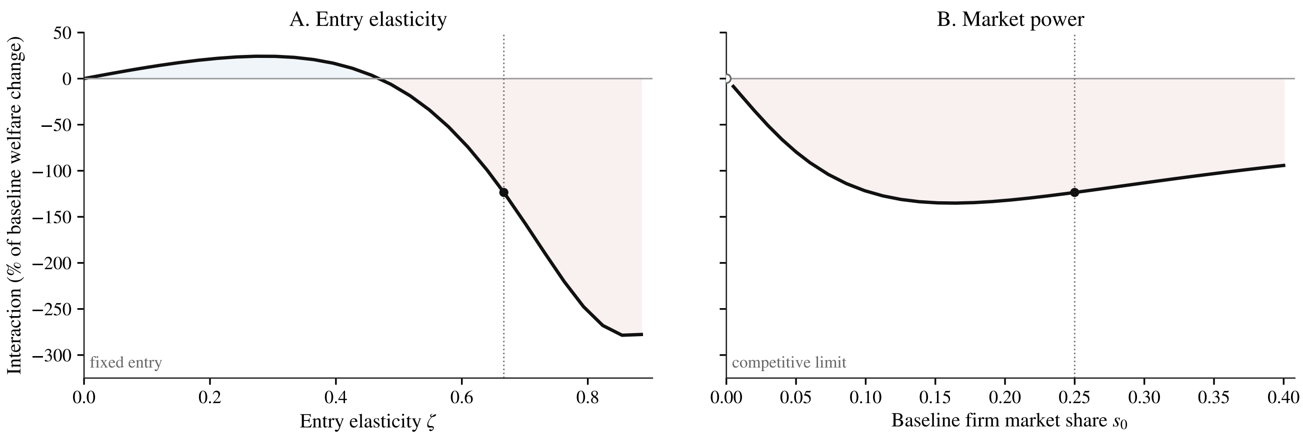


Figure 1: Exact interaction effects in the AB benchmark.

Notes: $\mathcal{J} = \Delta \log W^{11} - \Delta \log W^{10} - \Delta \log W^{01} + \Delta \log W^{00}$, where (00) is fixed markups and fixed entry, (10) is variable markups and fixed entry, (01) is fixed markups and variable entry, and (11) is variable markups and variable entry. The figure reports exact nonlinear interaction effects in the AB fixed-cost model with $\rho = \sigma = 10.5$, normalized by the baseline full-model welfare change for the 5% $\rightarrow$ 15% benchmark. Panel A varies entry elasticity and nests the exact fixed-entry benchmark at $\zeta = 0$. Panel B varies baseline firm market share. Dotted lines and black dots mark the baseline calibration. The hollow dot at $s_0 = 0$ marks the competitive limit.

4 General Welfare Decomposition

Section 3 provides analytical insight on why and how markup-entry interactions play a key role in driving the welfare effects of trade wars in a tractable two-country model. Specifically, markup-entry interactions matter for welfare by impacting the allocation of expenditure across origins, the price domestic producers charge relative to wages, and the distribution of income. In this section, we show that these three objects are sufficient to build an exact *ex post* welfare decomposition, applicable to a general environment (and thus to the rich baseline models we use in our quantitative analysis). In particular, we ask: after a tariff changes markups, firm survival, and production networks, what is the contribution of price movements and shifts in income and expenditure to the change in welfare?

We answer this question in the two propositions and the corollary below. Proposition 3 extends the welfare decomposition by Arkolakis, Costinot and Rodríguez-Clare (2012) to environments with tariffs, variable markups, production networks, and discrete entry and exit. Proposition 4 does the same for the welfare accounting framework by Baqaee and Farhi (2024). Corollary 2 reconcile the two, showing the connection between the two decompositions. In the text, we present the main conclusion in a readily interpretable forms—proofs and details are in Online Appendix B.3 and Appendix C.

Environment For a fixed destination d, the ACR welfare identity in Proposition 3 relies on two observable demand objects: sectoral domestic expenditure shares $\lambda_{dd,s}$ and sector weights $\theta_{ds} \equiv X_{ds}/E_d$, where X_{ds} is destination d's final expenditure on sector s and E_d is total final expenditure.

Let $t \in \{0,1\}$ index the initial and counterfactual equilibria, and let hats denote gross changes, $\hat{x} \equiv x_1/x_0$. Welfare is real final-demand expenditure, $W_{d,t} \equiv E_{d,t}/P^{FD}_{d,t}$, where $P^{FD}_{d,t}$ is destination d's final-demand price index. National income equals resident wage income, resident profit income, and rebated tariff revenue: $E_{d,t} = w_{d,t}L_{d,t} + \Pi_{d,t} + \mathrm{TR}_{d,t}$. Let $\tau_{\Pi,d,t} \equiv \Pi_{d,t}/E_{d,t}$ and $\tau_{\mathrm{TR},d,t} \equiv \mathrm{TR}_{d,t}/E_{d,t}$ denote the profit and tariff-revenue shares of income, let $\widetilde{P}_{dd,s,t} \equiv P_{dd,s,t}/w_{d,t}$ denote the price of the domestic sector-s bundle relative to the domestic wage, and let $\varepsilon \equiv 1-\rho < 0$. Tariffs therefore matter not only through trade shares and prices, but also through the composition of nominal income.

Exact comparison of two equilibria with entry and exit uses Sato–Vartia aggregation on continuing varieties together with Feenstra variety corrections. Online Appendix B.2 gives the exact Sato–Vartia and Feenstra formulas. The main-text results below use those exact index-number objects directly, so entry and exit are not treated as residuals or approximation errors.

4.1 An extended ACR welfare identity

The classical Arkolakis, Costinot and Rodríguez-Clare (2012) formula expresses welfare changes through one sufficient statistic: the domestic expenditure share raised to the power $1/\varepsilon$. In our environment, exact welfare accounting requires two additional blocks for the reasons highlighted in Section 3. A tariff still reallocates expenditure across origins (an effect captured by the original ACR), but it also changes domestic producer prices relative to wages through changes in markups, input costs, and the set of active firms. In addition, it also changes the composition of nominal income through profits and tariff revenue. The exact ACR-form identity we offer below therefore contains the classical trade-share term plus two blocks: a domestic-price block and an income block.

Proposition 3 (Extended ACR welfare identity)**.** *Let ω^{SV}_{ds} denote Sato–Vartia weights across* *sectors, constructed from the logarithmic mean of period-0 and period-1 sectoral expenditure shares* $\theta_{ds,t} \equiv X_{ds,t}/E_{d,t}$*.*[5] *Define effective domestic-share and domestic-price terms:*

$$\widehat{\lambda}_{dd}^{\text{eff}} \equiv \exp\Big(\sum_s \omega_{ds}^{SV}\,\Delta\log\lambda_{dd,s}\Big), \qquad \widehat{\widetilde{P}}_{dd}^{\text{eff}} \equiv \exp\Big(\sum_s \omega_{ds}^{SV}\,\Delta\log\frac{P_{dd,s}}{w_d}\Big).$$

Under the common-ρ CES benchmark used in our quantitative model, fixed labor supply, and exact sector aggregation on the common comparison set,

$$\hat{W}_d = \frac{1-\tau_{\Pi,d,0}-\tau_{\text{TR},d,0}}{1-\tau_{\Pi,d,1}-\tau_{\text{TR},d,1}}\big(\widehat{\widetilde{P}}_{dd}^{\text{eff}}\big)^{-1}\big(\widehat{\lambda}_{dd}^{\text{eff}}\big)^{1/\varepsilon}. \tag{24}$$

The three factors are, respectively, the income correction, the domestic-price correction, and the ACR term, with $\varepsilon \equiv 1-\rho < 0$*. In logs:*

$$\Delta\log W_d = \mathcal{A}_d^{\lambda} + \mathcal{A}_d^{E} + \mathcal{A}_d^{P}, \tag{25}$$

where

$$\mathcal{A}_d^{\lambda} \equiv \frac{1}{\varepsilon}\sum_s \omega_{ds}^{SV}\,\Delta\log\lambda_{dd,s}, \qquad \mathcal{A}_d^{E} \equiv \Delta\log\frac{1-\tau_{\Pi,d,0}-\tau_{\text{TR},d,0}}{1-\tau_{\Pi,d,1}-\tau_{\text{TR},d,1}}, \qquad \mathcal{A}_d^{P} \equiv \sum_s \omega_{ds}^{SV}\,\Delta\log\Big(\frac{w_d}{P_{dd,s}}\Big).$$

Proof. See Online Appendix B.3.1 (Proposition B.7 and Corollary B.1) for the full statement and proof.[6]

The ACR term $(\widehat{\lambda}_{dd}^{\text{eff}})^{1/\varepsilon}$ is the reallocation block. It records whether expenditure shifts toward or away from domestic producers, just as the origin-share movements S_D and S_F did in Section 2. If this were the only thing tariffs changed, classical Arkolakis, Costinot and Rodríguez-Clare (2012) would remain sufficient.

The domestic-price term $(\widehat{\widetilde{P}}_{dd}^{\text{eff}})^{-1}$ collects every force that moves the price of the domestic bundle relative to the domestic wage: markup adjustment, exact variety corrections from entry and exit, and network propagation through domestic input costs. The income correction is the nominal-income counterpart. It collects changes in resident claims on wages, profits, and tariff revenue. A tariff can therefore raise the domestic share and still lower welfare if it also raises domestic producer prices relative to wages or shifts income away from domestic residents. Those

[5]The logarithmic mean is $\mathcal{L}(a,b) \equiv (a-b)/(\log a - \log b)$ for $a \neq b$ and $\mathcal{L}(a,a) = a$. The Sato–Vartia weight is $\omega_{ds}^{SV} \equiv \mathcal{L}(\theta_{ds,0},\theta_{ds,1})/\sum_{s'}\mathcal{L}(\theta_{ds',0},\theta_{ds',1})$. Unlike Tornqvist (arithmetic-mean) weights, these deliver an exact additive decomposition of log CES price changes; see Online Appendix B.2.

[6]The Online Appendix states the exact theorem for any nominal anchor N_d, that is, any choice of destination-specific numeraire, and the sector-specific-ρ_s corollary. We report the common-ρ, fixed-labor ($N_d = w_d$) benchmark used in the quantitative exercises.

are exactly the two effects that sit outside the classical domestic-share term.

4.2 A BF-style decomposition

The next proposition offers a decomposition after Baqaee and Farhi (2024), extending their results to an environment with entry and exit of firms.

Proposition 4 (Extended BF welfare decomposition)**.** *Let* Tech_d*,* MarkupWedge_d*,* TariffWedge_d*,* FactorBuy_d*, and* VarietyPrice_d *denote the exact price-side channels constructed in Online Appendix B.3 from producer productivity, markups, tariff wedges, factor costs, network exposures, and exact variety corrections. Let* FactorSell_d *and* VarietyIncome_d *denote the corresponding exact income-side channels from continuing and discontinuous resident claims. Then welfare* $W_{d,t} \equiv E_{d,t}/P^{FD}_{d,t}$ *admits the exact decomposition*

$$\begin{aligned}\Delta \log W_d = & \underbrace{\text{Tech}_d + \text{MarkupWedge}_d + \text{TariffWedge}_d}_{\textit{producer-side wedges}} + \underbrace{\text{FactorBuy}_d + \text{FactorSell}_d}_{\textit{factor and income channels}} \\ & + \underbrace{\text{VarietyPrice}_d + \text{VarietyIncome}_d}_{\textit{variety channels}}. \end{aligned} \tag{26}$$

Equivalently, the welfare deflator and nominal-income pieces satisfy:

$$-\Delta \log P^{FD}_d = \text{Tech}_d + \text{MarkupWedge}_d + \text{TariffWedge}_d + \text{FactorBuy}_d + \text{VarietyPrice}_d, \tag{27}$$

$$\Delta \log E_d = \text{FactorSell}_d + \text{VarietyIncome}_d. \tag{28}$$

Proof. See Online Appendix B.3.5 (Proposition B.13) for the detailed definition of each channel, and Appendix C for the proofs.

Proposition 4 identifies seven channels through which tariffs affect welfare. The first five drive changes in destination d's welfare price index. The last two drive changes in destination d's nominal expenditure. If a tariff induces exporter exit, the price index can worsen because surviving exporters raise markups (MarkupWedge_d) and because some goods disappear (VarietyPrice_d). At the same time, nominal expenditure can change because those same exit decisions destroy profit claims (VarietyIncome_d) or reallocate profits among continuing firms (FactorSell_d). The theorem therefore decomposes welfare changes into price distortions in continuing trade relationships, changes in the set of firms serving the market, and changes in the allocation of income. Entry and exit move the price index through VarietyPrice_d and nominal expenditure through VarietyIncome_d. The profit component of FactorSell_d captures how the income of continuing firms changes when sales and markups reallocate across producers.

4.3 A bridge across decompositions

Corollary 2 connects two theoretical results in the two Propositions 3 and 4, and bridges the detailed BF decomposition channels to the three ACR blocks.[7]

Corollary 2 (Mapping BF decomposition to ACR identity). *Under Propositions 3–4, the ACR blocks of Proposition 3 map to channels as:*

$$\mathcal{A}_d^{\lambda} + \mathcal{A}_d^{P} = \text{Tech}_d + \text{MarkupWedge}_d + \text{TariffWedge}_d + \text{FactorBuy}_d + \text{VarietyPrice}_d + \text{WageNum}_d, \tag{29}$$

$$\mathcal{A}_d^{E} = \text{FactorSell}_d + \text{VarietyIncome}_d - \text{WageNum}_d, \tag{30}$$

where $\text{WageNum}_d \equiv \Delta \log w_d$ *is the change in d's wage numeraire, which appears with opposite signs in the price and income blocks and cancels in total:*

$$\Delta \log W_d = (\mathcal{A}_d^{\lambda} + \mathcal{A}_d^{P}) + \mathcal{A}_d^{E} = \sum_{k \in \mathcal{K}} \text{Channel}_{k,d}. \tag{31}$$

where $\mathcal{K}$ *indexes the seven channels listed in Proposition 4.*

Proof. See Online Appendix B.3.6 (Proposition B.14 and Corollary B.5).

We conclude noting that the results in Section 3 are a special case of Propositions 3–4, with one sector and no production-network propagation. In that case, Tech_d and FactorBuy_d vanish, and the interaction term arises because entry simultaneously changes the markup channels and the variety channels. Online Appendix B.1.6 provides the corresponding closed-form analysis.

5 Quantitative Model

In building our quantitative model, we pursue the following three objectives: matching the granular market structure in the data, capturing the effects of tariffs on both markups and participation, and tracing adjustments through production networks. We discipline the model using aggregate trade shares together with firm-level tariff elasticities.

We model a world consisting of a set of countries $\mathcal{C}$, with trade indexed by origin $o \in \mathcal{C}$ and destination $d \in \mathcal{C}$.[8] Each country produces a continuum of tradable products indexed by i, that can

[7]Online Appendix Corollary B.5 further separates the combined price block into the domestic-price correction $\mathcal{A}_d^{P}$ and the reallocation term $\mathcal{A}_d^{\lambda}$.

[8]Throughout our paper, we use calligraphy math symbols to indicate a set of elements.

be used in either final consumption or as intermediate inputs.[9] In the quantitative implementation, we approximate that continuum of products with a finite product grid. Most crucially, we model oligopolistic markets at the firm level in both final-good and intermediate-input sales.

5.1 Preferences, Technology and Market Structure

As shown in Crowley, Han and Prayer (2024), the empirical evidence on the pro-competitive effects of tariffs lends empirical support to demand systems that recognize that, for a given product, the elasticity of substitution may be higher for varieties produced in the same origin, than for varieties originating in different countries. As in that paper, we model a demand system nesting three layers, in both final demand and in intermediate-input demand. Products aggregate into the destination basket, origins aggregate within a product, and firms aggregate within an origin—with different elasticities. Specifically, η governs substitution across products, ρ governs substitution across origins within a product, and σ governs substitution across firms from the same origin.

Final consumption Y_{dt} and the price of the final consumption good P_{dt} in each country d in period t are aggregated over products i used in final demand:

$$Y_{dt} = \left(\int_{i \in \mathcal{FD}} (\alpha_{id})^{\frac{1}{\eta}} y_{idt}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}, \quad P_{dt} = \left(\int_{i \in \mathcal{FD}} \alpha_{id} p_{idt}^{1-\eta} di \right)^{\frac{1}{1-\eta}} \tag{32}$$

where $\eta > 1$ is the elasticity of substitution across products, $\mathcal{FD}$ is the set of products used for final demand, and α_{id} is a demand shifter at the product-destination level.

For a given product i, demand aggregates origin-specific bundles into the product-level quantity y_{idt} and price index p_{idt}:

$$y_{idt} = \left(\sum_{o \in \mathcal{C}} (\alpha_{od})^{\frac{1}{\rho}} y_{iodt}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad p_{idt} = \left(\sum_{o \in \mathcal{C}} \alpha_{od} p_{iodt}^{1-\rho} \right)^{\frac{1}{1-\rho}} \tag{33}$$

where $\rho \geq \eta$ is the elasticity of substitution across origins within a product, and α_{od} is a demand shifter at the origin-destination level.

Within each product-origin-destination triplet, a finite set of firms supplies differentiated varieties. The origin-specific bundle y_{iodt} and its price index p_{iodt} are

$$y_{iodt} = \left(\sum_{f \in \mathcal{F}_{iodt}} (\alpha_{fiodt})^{\frac{1}{\sigma}} y_{fiodt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad p_{iodt} = \left(\sum_{f \in \mathcal{F}_{iodt}} \alpha_{fiodt} p_{fiodt}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{34}$$

[9] In Section 2, the empirical counterpart of a product is a consolidated HS6 category. In the quantitative model, we aggregate those products into broader WIOD-based categories and approximate each category with a finite product grid. We use "product" and "industry" interchangeably when no confusion arises.

where $\sigma \geq \rho$ is the elasticity of substitution across varieties from the same origin, α_{fiodt} is an idiosyncratic demand shifter within the origin-product-destination nest, and $\mathcal{F}_{iodt}$ represents the set of active firms that sell product i from origin o to destination d at time t.[10]

The intermediate-input bundle M_{dt} uses the same nesting order and elasticities:

$$M_{dt} = \left(\int_{i \in \mathcal{IM}} (\alpha_{id}^{M})^{\frac{1}{\eta}} m_{idt}^{\frac{\eta-1}{\eta}} \, di \right)^{\frac{\eta}{\eta-1}} \tag{35}$$

where $\mathcal{IM}$ is the set of products used for intermediate input, and α_{id}^{M} is a demand shifter at the product-destination level for intermediate inputs. Let P_{dt}^{M} denote the associated dual price index.

Within each product, aggregation across origins and firms follows:

$$m_{idt} = \left(\sum_{o \in \mathcal{C}} (\alpha_{od}^{M})^{\frac{1}{\rho}} m_{iodt}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad m_{iodt} = \left(\sum_{f \in \mathcal{F}_{iodt}^{M}} m_{fiodt}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{36}$$

where α_{od}^{M} is a demand shifter at the origin-destination level for intermediate inputs, and $\mathcal{F}_{iodt}^{M}$ is the set of active firms producing intermediate inputs. For brevity, we suppress the corresponding lower-level dual price indices. The economic point is unchanged: an intermediate-input seller faces the same share-weighted perceived elasticity as in final demand.

Production Technology. Labor is inelastically supplied and immobile across countries, and wages are identical across sectors within a country. Output is produced with a Cobb-Douglas technology in labor and intermediate inputs. Tariffs therefore affect marginal cost both directly through trade wedges and indirectly through the cost of intermediate inputs. The output of a firm with productivity A_{fiot} is

$$q_{fiot} = A_{fiot} \left(\frac{L_{fiot}}{\nu} \right)^{\nu} \left(\frac{M_{fiot}}{1-\nu} \right)^{1-\nu} \tag{37}$$

where q_{fiot} is total quantity produced with L_{fiot} labor and M_{fiot} units of the intermediate input bundle; and ν is the labor share in production. For a final-good producer, total output satisfies $q_{fiot} = \sum_d y_{fiodt}$; for an intermediate-input supplier, $q_{fiot} = \sum_d m_{fiodt}$.

The implied marginal cost is

$$mc_{fiot} = \left(\frac{W_{ot}}{\nu} \right)^{\nu} \left(\frac{P_{ot}^{M}}{1-\nu} \right)^{1-\nu} / A_{fiot},$$

where W_{ot} is the origin-country wage and P_{ot}^{M} is the price of the intermediate-input bundle. The intermediate-input price is the channel through which upstream tariff shocks propagate into down-

[10] We indicate a variable's level of aggregation in our model by its subscript. The most disaggregated variables have five dimensions, f, i, o, d and t, which stand for firm, industry, origin, destination, and time, respectively.

stream producers.

5.2 Firm's problem

Below we write the destination-specific problem for a final-good seller. The intermediate-input seller faces the same problem, with the intermediate-input demand block replacing the final-demand block. Firms first decide whether to serve a market and then choose sales conditional on entry. Because marginal cost is constant, the problem is separable across destinations. Firm f's per-period profit from selling product i from origin o in destination d is

$$\pi_{fiodt} = \left[y_{fiodt} \left(\frac{p_{fiodt}}{\tau_{iodt}} - \iota_{iodt} mc_{fiot} \right) - \zeta_x \right] \phi_{fiodt},$$

where demand is

$$y_{fiodt} = \alpha_{id} \alpha_{od} \alpha_{fiodt} \left(\frac{p_{fiodt}}{p_{iodt}} \right)^{-\sigma} \left(\frac{p_{iodt}}{p_{idt}} \right)^{-\rho} \left(\frac{p_{idt}}{P_{dt}} \right)^{-\eta} Y_{dt} \tag{38}$$

where τ_{iodt} and ι_{iodt} are the gross ad-valorem tariff and non-tariff trade-cost wedges for firms in origin o to sell product i in destination d at time t.[11] ζ_x is a constant per-period export cost in terms of final consumption units. A firm serves destination d only if its operating profit in that market, $y_{fiodt}(p_{fiodt}/\tau_{iodt} - \iota_{iodt} mc_{fiot})$, covers the fixed per-period exporting cost ζ_x.[12] To select the active set when multiple participation profiles satisfy zero-profit conditions, we assume that firms enter sequentially in reverse order of marginal costs.[13] This rule picks the equilibrium that keeps the lowest-cost firms active and removes multiplicity created by discrete participation. We calibrate fixed operating costs so that markets open to trade are served by many suppliers in total, but only a handful of firms from the same origin enter a given product-destination market. That is the granular environment in which a single exit can move incumbent market shares by a nontrivial amount.

Conditional on entry, Cournot competition implies that the optimal price p_{fiodt} satisfies the standard markup rule, with the endogenous (destination-specific) markup μ_{fiodt} applied to the

[11] The tariff wedge is embedded in the purchaser price but is not received by the firm. We define $p^b_{fiodt} = p_{fiodt}/\tau_{iodt}$ as the border price before firm f's export of product i from origin o arrives at the border of destination d at time t. We model the non-tariff wedge ι_{iodt} as a Hicks-neutral productivity shifter for firms in origin o selling product i in destination d at time t.

[12] The production and pricing decisions for a firm selling in its own domestic (origin) market are similarly defined with a smaller fixed cost of operating in the domestic market, $\zeta_h < \zeta_x$, and bilateral tariff and trade costs normalized to one ($\tau_{ioot} = 1$, $\iota_{ioot} = 1$).

[13] This selects the equilibrium in which the most efficient firms operate whenever multiple active sets satisfy the zero-profit conditions.

tariff- and trade-cost-inclusive marginal cost mc_{fiodt}:

$$p_{fiodt} = \mu_{fiodt} mc_{fiodt}, \quad \mu_{fiodt} \equiv \frac{\varepsilon_{fiodt}}{\varepsilon_{fiodt} - 1}, \quad mc_{fiodt} \equiv mc_{fiot} \iota_{iodt} \tau_{iodt} \tag{39}$$

where ε_{fiodt} is the perceived demand elasticity under Cournot competition, given by equation (40). The same pricing rule applies to intermediate-input suppliers, with the relevant market shares computed inside the intermediate-input nesting tree.

Demand elasticity. Under Cournot competition, perceived demand elasticities depend on (i) the elasticities of substitution across products, origins, and firms, and (ii) the distribution of market shares both within and across origins. Under the assumption of the nested demand structure in equations (32)–(34), the perceived demand elasticity takes the form:[14]

$$\varepsilon_{fiodt} = \left[\frac{1}{\sigma} + \left(\frac{1}{\rho} - \frac{1}{\sigma}\right) ms_{fiodt} + \left(\frac{1}{\eta} - \frac{1}{\rho}\right) ms_{fiodt} ms_{iodt}\right]^{-1} \tag{40}$$

where the first market share ms_{fiodt} captures the importance of the firm among all exporters from its origin and the second market share ms_{iodt} captures the importance of the origin country in the destination market:

$$\underbrace{ms_{fiodt} = \frac{p_{fiodt} y_{fiodt}}{\sum_{f' \in \mathcal{F}_{iodt}} p_{f'iodt} y_{f'iodt}}}_{\text{firm's within-origin market share}}, \qquad \underbrace{ms_{iodt} = \frac{p_{iodt} y_{iodt}}{\sum_{o' \in \mathcal{C}} p_{io'dt} y_{io'dt}}}_{\text{origin's market share in the destination}} \tag{41}$$

Special cases. Equation (40) nests several benchmark cases:

$$\varepsilon_{fiodt} = \begin{cases} \sigma, & \text{if } ms_{fiodt} \to 0 \text{ or } \sigma = \rho = \eta, \\ \left[\frac{1}{\sigma} + \left(\frac{1}{\eta} - \frac{1}{\sigma}\right) \omega_{fiodt}\right]^{-1}, & \text{if } \sigma = \rho, \\ \left[\frac{1}{\sigma} + \left(\frac{1}{\rho} - \frac{1}{\sigma}\right) ms_{fiodt}\right]^{-1}, & \text{if } ms_{iodt} \to 0, \end{cases} \tag{42}$$

where $\omega_{fiodt} \equiv ms_{iodt} \cdot ms_{fiodt}$ is the firm's destination-level expenditure share. When firms are atomistic, or when all three elasticities coincide, markups are constant at $\sigma/(\sigma - 1)$ and trade-cost changes pass through one-for-one into prices. When $\sigma = \rho$, the within-origin and across-origin nests collapse and the model reduces to the Atkeson and Burstein (2008) benchmark, in which markups depend on the firm's destination-level share ω_{fiodt}. In that case, a lower destination-

[14]Consistent with the literature, we use Cournot competition as our benchmark since it better matches the relationship between pass-through and market shares (Atkeson and Burstein 2008; Amiti, Itskhoki and Konings 2019). Online Appendix B.1.2 derives the Cournot formula from the nested CES system and reports the Bertrand counterpart.

level expenditure share implies a lower markup, holding the active set fixed. When $ms_{iodt} \to 0$, competition from other origins becomes negligible and markups depend only on the within-origin share ms_{fiodt}. In that limit, exit of same-origin rivals raises survivors' shares and can increase their markups even while the origin loses market share in the destination.

The nested elasticity model highlights two key margins through which a tariff affects markups. First, holding within-origin shares fixed, a higher tariff lowers the share of firms from the targeted origin in the destination market ms_{iodt}. Since $\eta \leq \rho$, this raises the perceived elasticity faced by exporters from that origin and tends to lower their markups. Second, the exit of firms induced by the tariff raises the shares of continuing firms from the same origin ms_{fiodt}. When $\sigma > \rho$, the coefficient $(1/\rho - 1/\sigma)$ in equation (40) is positive, so that within-origin reallocation lowers perceived elasticities and tends to raise the markups of continuing firms. The net markup response is determined by the relative strength of these two forces.

Our quantitative model allows a small, discrete set of active suppliers in each market. Entry and exit caused by trade policy change the market shares of continuing firms by discrete amounts. A discrete reallocation is the source of the quantitative relevance of the interaction term discussed above.

5.3 Calibration

We calibrate the model to match two sets of moments. The first is baseline expenditure shares in final and intermediate use. The second is the set of firm-level tariff elasticities estimated in the data. We aim to simultaneously discipline the aggregate structure of trade flows and the micro-level responses of firms to tariff changes. This joint calibration is computationally demanding because, for any given set of aggregate conditions, the general equilibrium requires every firm's entry, exit, and pricing decisions—with strategic interactions under oligopoly—to converge before trade shares and tariff elasticities can be evaluated.

The calibration alternates between two phases. In *Phase 1*, for a given structural parameter vector, we update demand shifters until the model matches the targeted baseline shares up to the tolerances reported in the Online Appendix. In *Phase 2*, holding those shifters fixed, we update the structural parameter vector $(\rho, \sigma, \xi, \varsigma)$ by simulated method of moments (SMM), minimizing the weighted absolute deviation between model-implied and empirical tariff elasticities. The alternation repeats until both the share fit and the parameter vector stabilize.[15]

Phase 1: Baseline shares and production network. We calibrate the demand shifters $(\alpha_{id}, \alpha_{od}, \alpha_{id}^M, \alpha_{od}^M)$ to match baseline expenditure shares in final-demand and intermediate-input use in the 2014 World Input-Output Database (WIOD). For computation, we aggregate 45 countries

[15]See Appendix E.1 for the numerical algorithm and Appendix E.2 for the industry classification and calibration details.

into 6 groups: US, China, Canada, Mexico, EU, and ROW. We also aggregate 56 industries into 9 categories[16] and approximate each category with 50 products, yielding 450 products in total.[17]

By construction, this step produces a near-exact fit at the origin-destination and industry-destination levels and a reasonable fit at the more granular industry-origin-destination level (Figure 2).

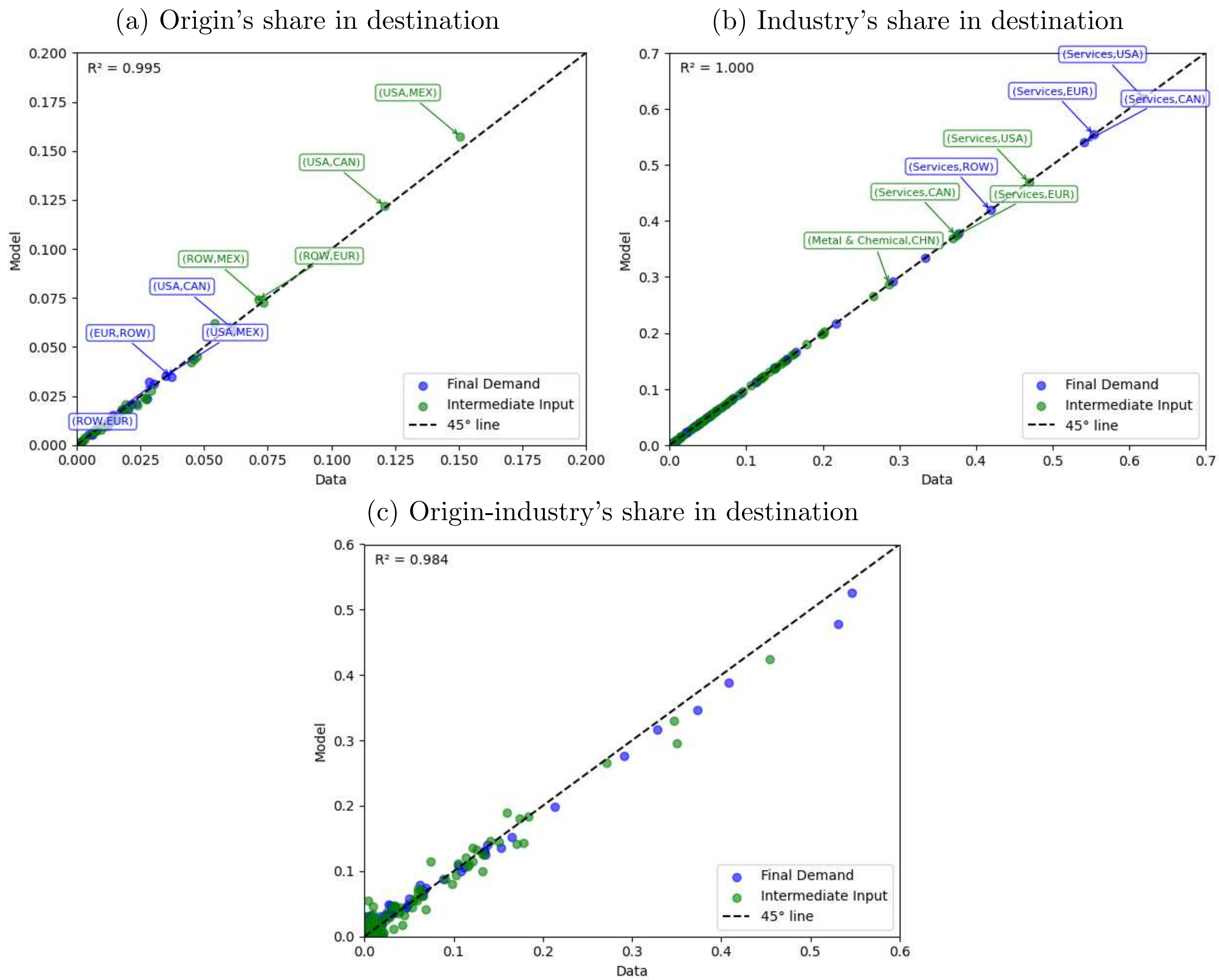


Figure 2: Model's fit to trade shares

Notes: Each panel plots model-predicted against observed trade shares. Panels (a) and (b) are the targeted origin-destination and industry-destination margins and therefore line up almost exactly. Panel (c) shows the untargeted industry-origin-destination margin.

In panels (a) and (b) in Figure 2 the shares are targeted directly, and thus these figure validate

[16] The 9 industry categories are: (1) Agriculture & Natural Resources, (2) Food, Textiles & Basic Manufacturing, (3) Metals & Chemical Manufacturing, (4) Electronics & Machinery, (5) Transport Equipment Manufacturing, (6) Other Manufacturing & Repair, (7) Utilities & Construction, (8) Wholesale, Retail & Transportation Services, (9) Knowledge, Public & Personal Services.

[17] For each product-country pair, the quantitative implementation allows 10 potential firms.

the numerical fit of our target. The untargeted panel (c) shows the fit for the the full industry-origin-destination variation, showing the model performs well in all data dimensions.

Our calibration approach differs from existing methods in that it both matches ex-ante trade shares and traces endogenous changes in markups and market shares. Baqaee and Farhi (2024) take ex-ante trade shares directly from the data as primitives, but assumes exogenously given wedge (markup and tariff) changes to solve for the new equilibrium. Mukhin (2022) pin down ex-ante shares from the data with Cobb-Douglas demand shifters, but this rule out any ex-post adjustments. Ferrante, Graves and Iacoviello (2023) allow ex-post changes through CES aggregation, but do not guarantee that ex-ante shares match the data because demand shifters are set equal to trade shares. By calibrating separate demand shifters for final goods and intermediate inputs at multiple levels of aggregation, we replicate observed production networks while allowing for realistic responses to policy changes.

Phase 2: Firm-level tariff elasticities. Given the demand shifters from Phase 1, we use SMM to choose structural parameters that minimize the weighted absolute deviations between model-implied and empirical tariff elasticities estimated in Crowley, Han and Prayer (2024). The targeted moments are the destination-specific value and markup elasticities, the within-origin and origin market-share elasticities, and the elasticity of the number of active exporters. Table 2 implies $\sigma > \rho$, so firms compete more strongly with exporters from the same origin than with exporters from other origins. We also choose the fixed-cost block so that the average foreign-market participation rate is 25%. The domestic fixed operating cost is set to 10% of the export fixed operating cost, which keeps domestic participation close to universal while export participation remains selective.[18]

Identification intuition. The five moments load on different parts of the mechanism. The two market-share elasticities discipline how tariffs reallocate demand across origins and within an origin. The markup elasticity then tells us whether that reallocation is strong enough to move perceived elasticities in the direction observed in the data. Matching a positive markup elasticity together with a negative origin-share elasticity and a positive within-origin-share elasticity requires same-origin competition to be stronger than cross-origin competition, so the calibration pushes the model toward $\sigma > \rho$. The extensive-margin elasticity and the participation target then pin down the dispersion of firm heterogeneity together with the fixed-cost block.

[18] In the quantitative implementation, fixed export costs are re-calibrated within the outer loop whenever the share of active exporters moves materially away from the 25% target; see Appendix E.2.

Table 2: Calibration: estimated parameters and targeted moments

Panel A: Estimated parameters

	Value
Cross-origin elasticity of substitution (ρ)	3.51
Within-origin elasticity of substitution (σ)	7.32
Pareto shape, idiosyncratic productivity (ξ)	11.97
Dispersion, idiosyncratic demand (ς)	0.85

Panel B: Targeted moments

	Data	Model
Value elasticity	−0.78	−0.87
Markup elasticity	0.41	0.23
Within-origin market share elasticity	2.88	2.34
Origin market share elasticity	−3.67	−3.20
Number of firms elasticity	−2.45	−2.23

Notes: Panel A reports parameters estimated by SMM. The cross-product elasticity of substitution (η) is exogenously set to 1.2, close to values in Atkeson and Burstein (2008) and Edmond, Midrigan and Xu (2015). Each product-country pair has 10 potential firms drawing productivity from a Pareto distribution with CDF $F_x(x) = 1 - x^{-\xi}$ for $x \geq 1$. The demand-preference dispersion ς governs the firm-level demand shifter: $\log \alpha_{fiodt} \sim \mathcal{N}(0, \varsigma^2)$. Panel B reports tariff elasticities from Crowley, Han and Prayer (2024). Each data coefficient is from a regression of the outcome on $\ln(1 + \tau_{iodt})$ with origin-firm-product-year, industry-destination-year, and origin-destination fixed effects; the number-of-firms regression uses PPML with origin-industry-year, destination-industry-year, and origin-destination fixed effects.

Taken together, the estimated elasticities imply a quantitatively strong within-origin competition margin. Because σ is roughly twice as large as ρ, a tariff can lower an origin's overall destination share while still raising the markups of surviving exporters from that origin. This is key in our calibration, as the difference in competition intensity implies that the extensive margin feeds back also into market power, rather than only into variety. Our model matches the sign pattern and broad magnitudes of the five targeted tariff elasticities in Crowley, Han and Prayer (2024).

Beyond the targeted elasticities, the model reproduces the broad distribution of within-origin concentration documented in Table 1. Table 3 compares model-implied concentration statistics with their data counterparts. The model captures the high concentration of granular export markets: the Herfindahl-Hirschman Index and top-two share suggest that within-origin sales are dominated by a handful of firms. Conditional on cells with at least one incumbent and one entrant, the model also generates entrant shares of the right order of magnitude.

Table 3: Untargeted moments: within-origin export concentration

	25th Percentile	Median	75th Percentile
(i) Herfindahl-Hirschman Index	0.36	0.52	1.00
[Data target]	0.35	0.65	1.00
(ii) Top-2 market share	76.8%	100.0%	100.0%
[Data target]	75.0%	99.6%	100.0%
(iii) Cumulative market share cond. on ≥ 1 incumbent and ≥ 1 entrant			
Incumbents	60.2%	71.6%	80.3%
[Data target]	31.2%	63.1%	86.7%
Entrants	19.7%	28.4%	39.8%
[Data target]	13.3%	36.9%	68.8%

Note: Concentration is measured at the product–origin–destination level. Model moments are computed from simulated data pooling all destinations. Data moments correspond to Panel (a) of Table 1, measured at the consolidated-HS6-product–origin–destination–year level using the universe of firms exporting from 11 origins to 165 destinations. Panel (iii) conditions on cells with at least one incumbent and one entrant, where incumbents sell in both periods and entrants sell in the current period only.

6 Reallocation of Market Power in a U.S.-Centered Trade War

We simulate a US-centered trade war in the calibrated world economy and use the four configurations from Sections 3 and 4 to dissect the impact of markup adjustment, entry and exit, and the interaction of the two.

6.1 Experiment Design

We simulate a trade war experiment in a world with six countries: US, China, Mexico, Canada, the EU, and rest of the world (ROW). Before the trade war, countries trade under the baseline tariffs reported in Table 4, which is calibrated to average bilateral tariffs in 2014. Tariffs among the NAFTA partners are near zero, while China and ROW face higher import tariffs on average. In the counterfactual experiment, the US–China bilateral tariff rises to 30% in both directions, and bilateral tariffs between the US and each of Mexico, Canada, the EU, and ROW rise to 10% in both directions. Tariffs between non-US country pairs remain at their baseline values.

The structure of the tariff shock also creates heterogeneity across countries that underpins the differential welfare impact of trade war. The US will be the only country that raises tariffs against every partner, while China is a large and closed economy facing a bigger shock; Mexico and Canada are small open economies tightly linked to the US through both final and intermediate trade; and the EU and ROW are less connected with the US through production networks.

Table 4: Pre-shock baseline tariffs (%, origin × destination)

	USA	CHN	MEX	CAN	EU	ROW
USA	—	7.0	0.2	0.1	2.7	6.4
CHN	2.9	—	3.7	2.9	1.8	4.2
MEX	0.0	7.9	—	0.0	0.1	6.6
CAN	0.0	4.7	1.0	—	1.8	4.8
EU	2.1	9.2	0.2	1.8	—	5.2
ROW	2.0	4.7	4.2	1.7	1.2	—

Notes: Each entry is the ad valorem tariff rate that the destination (column) charges on imports from the origin (row). Tariffs are calibrated to observed bilateral rates in 2014.

To bring our analysis of markup and entry interactions to bear on our quantitative results, we solve the model under four configurations, and present results in line with the welfare decompositions discussed in Section 4. Configuration (1), fixed entry and fixed markups (FE–FM), holds both the set of active firms and markups at their pre-shock values, so to capture only the direct reallocation of expenditure shares; this is the analogue of an ACR-style benchmark. Configuration (2), fixed entry and variable markups (FE–VM), allows continuing firms to re-optimize markups but freezes the active set, isolating the pure markup channel. Configuration (3), variable entry and fixed markups (VE–FM), allows firms to enter or exit but holds markups fixed, isolating the pure entry channel.[19] Configuration (4), variable entry and variable markups (VE–VM), is

[19] In VE–FM, markups of firms that were active in the pre-shock equilibrium are frozen at their pre-shock values. For firms that enter post-shock from an origin that was already present in the market, the frozen markup is the hypothetical markup implied by the pre-shock equilibrium: the within-origin share ms_{fiodt} is the share the firm would hold given its productivity and the pre-shock price index of its origin group, and the origin share ms_{iodt} is the origin's actual pre-shock share in the destination. These two shares pin down the perceived elasticity via equation (40) and therefore the markup via equation (39). For firms entering from an origin with no pre-shock presence, the origin share is zero and the within-origin share is one, so the perceived elasticity collapses to ρ and the frozen markup is $\rho/(\rho-1)$.

the full model in which both margins adjust jointly. In each of these, we report the interaction between markup adjustment and entry/exit, defined as the difference-in-differences

$$\text{Interaction} = \Delta \log W^{(4)} - \Delta \log W^{(2)} - \Delta \log W^{(3)} + \Delta \log W^{(1)}.$$

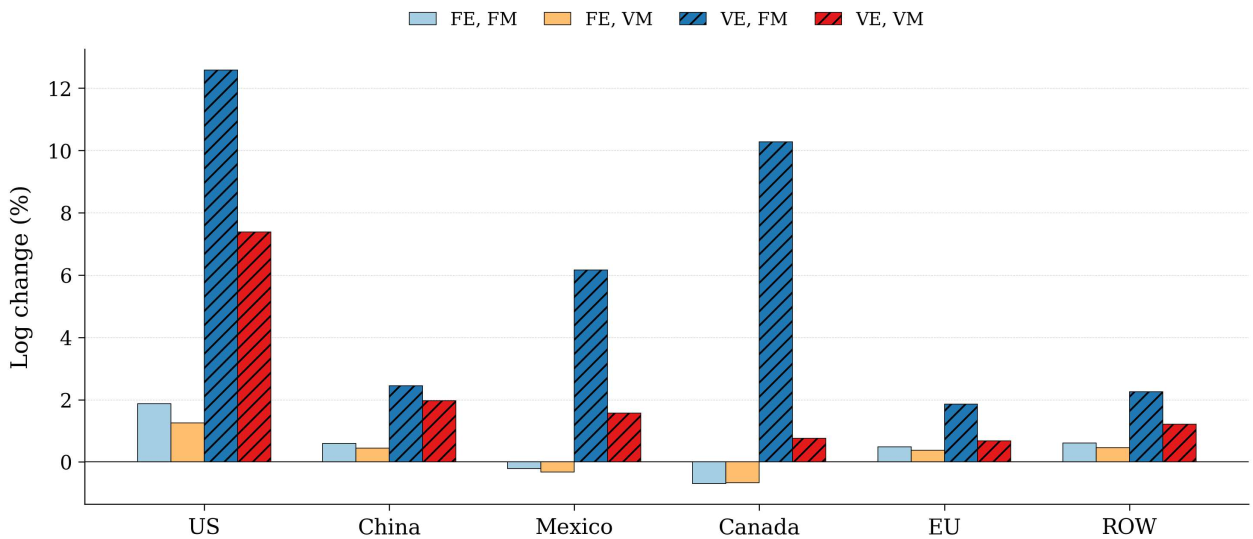
As shown analytically in Section 2, this interaction is nonzero whenever the two margins are complementary.

6.2 Concentration and Welfare

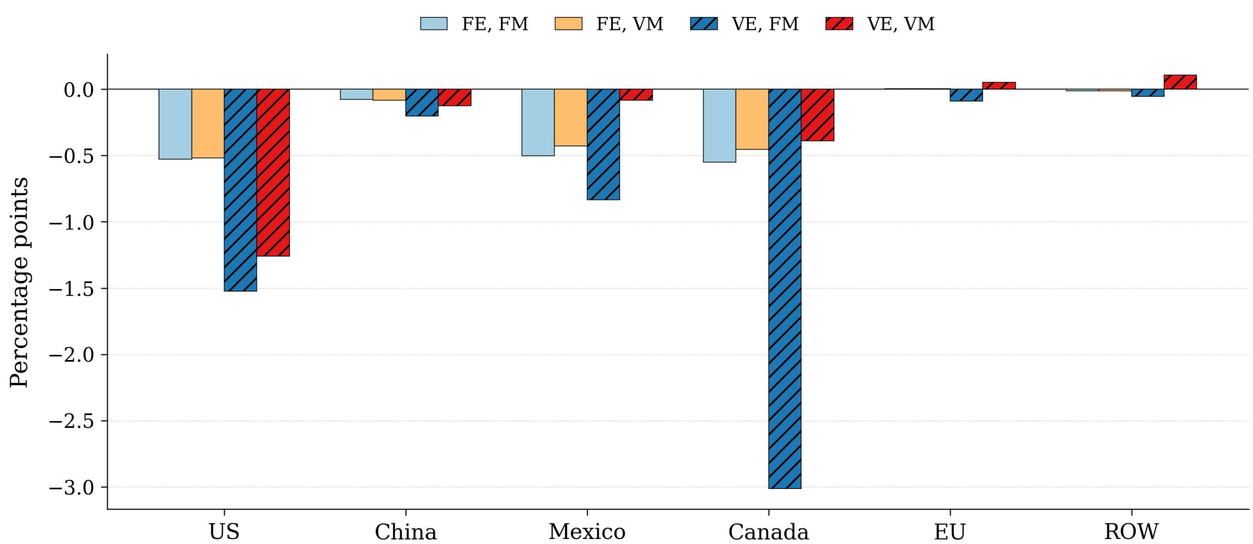
The point of departure of our analysis is an examination of the effect of the trade war on market concentration and welfare at country level. Figure 3a reports the log change in the Herfindahl–Hirschman index (HHI) in tradable sectors under the four configurations. Under fixed entry, concentration rises only modestly because reallocation occurs within a fixed set of active suppliers. With an endogenous response at the extensive margin, but no adjustment in markup, entry and exit leads to a sharp rise in concentration: as tariffs push foreign exporters out, sales concentrate among fewer sellers. Re-optimization of markups however attenuates this result, because some firms that would exit at the pre-shock markup can actually remain active in their export markets by repricing.

For the U.S., our model suggests that, in response to the trade war we parameterized in our baseline exercise for the full model (VE–VM), the rise in the concentration index in the US is of the order of 7 percent in log HHI. To put the magnitude of this increase in concentration in perspective, it is useful to compare it with the secular rise in concentration documented by a large empirical literature. By way of example, Grullon, Larkin and Michaely (2019) show that the Herfindahl–Hirschman index across U.S. industries rose approximately 70 percent between its trough in the late 1990s and 2014, with over 75 percent of industries experiencing an increase. Our result amounts to roughly one-tenth of this secular increase, that unfolded over nearly two decades.

Figure 3b unveils a striking association between rising concentration and welfare losses. Across all our configurations, countries that experience a sharp rise in concentration also suffer large welfare losses. In the fixed entry cases, the rise in concentration is limited, so is the welfare change due to trade war. With variable entry but fixed markup, the increase in concentration results in a significant reallocation of market share from foreign firms to domestic firms, which can then charge higher markups, driving down welfare. Adding markup adjustment has an additional crucial effect on the market structure, with more foreign firms remaining active. This attenuates both the rise in concentration and the associated welfare losses.



(a) Change in HHI in tradable sectors ($\Delta \log \text{HHI}$, %)



(b) Welfare change ($\Delta \log W$, percentage points)

Figure 3: Concentration and welfare effects of a US-centered trade war by country and model configuration. FE = fixed entry; VE = variable entry; FM = fixed markups; VM = variable markups.

6.3 Aggregate Welfare Decomposition

Before turning to a quantification of the welfare decomposition in Section 4, it is useful to recall its analytical structure. The welfare change $\Delta \log Y$ can be decomposed into an intensive margin—reallocation across continuing firms—and an extensive margin—entry and exit. Each margin has a price side and an income side. At the intensive margin, the income side of the decomposition captures how firms allocate revenues among factors of production: wages, profits, and tariff revenue from continuing relationships; the price side captures input costs, explicitly accounting for allocative wedges from markups and tariffs. At the extensive margin, the income side records the net flows of wage bills, profits, and tariff revenue attached to entering or exiting firms; the price side records the variety effect on the consumer price index. Within each margin, the price side further distinguish between domestic versus foreign; the income side betweem domestic sales versus export sales.

This layered structure allows us to investigate complementary and offsetting forces active at different levels. Namely, first, exit of foreign firms destroys varieties at the extensive margin but frees market share for continuing domestic firms allowing them to raise sales at the intensive margin, so the two margins move in opposite directions. Second, within the intensive margin, the expansion of domestic sales raises income but is accompanied by rising domestic markups that push up prices—creating a tension between income gains and losses from rising consumer prices. Third, looking at the income side, the gains in domestic sales on the intensive-margin are partly offset by losses in export sales on the extensive-margin, as a trade war that redirects demand toward domestic producers also induces exit from foreign markets. The quantitative question is which side of these trade-offs dominates.

Table 5 reports the welfare change ($\Delta \log Y$) for each country under our four configurations, with and without the production network.

Focusing first on the results from the full model (VE–VM), it is apparent that the welfare impact of the trade war will be quite heterogeneous across countries. As shown in the table, the US suffers the largest welfare loss because it is the only country raising tariffs against every trading partner, causing high foreign exit and the largest rise in concentration. China's loss is modest despite facing the hike in bilateral tariffs with the US, because the share of its trade hit by the tariff is small relative to its economy. Canada and Mexico are more exposed through their dependence on US trade flows and supply chains. The EU and ROW experience no significant welfare changes. Importantly, input-output linkages greatly amplify the welfare response – production networks scale up both the positive reallocation at the intensive margin and the negative variety effects.

Comparing across configuration illustrates the offsetting effects at the intensive and extensive margin. Under fixed entry (FE–FM, FE–VM), the extensive margin is muted, and the intensive-margin change from the markup adjustment is "elusive": in response to a tariff shock, foreign firms reduce their markups, but the effect on prices is largely offset higher markups charged by domestic firms. In addition, since the set of active firms is fixed, the intensive margin operates mainly through reallocation of expenditure across continuing firms, which is small in the absence of entry and exit. The net effect on welfare is a contained loss for every country.

When entry and exit respond to tariff shocks but markups are fixed (VE–FM), the intensive margin turns positive in every country (more so with the production network) because the reallocation of expenditure to domestic producers following the exit of (large) foreign firms can be significant. The flip side is a large loss in product varieties: on the price side, exit destroys varieties and raises the consumer price index; on the income side, firm exit reduces the wage bills and profits. In all countries, under variable entry and fixed markups (VE–FM) the loss of varieties more than offsets the intensive-margin gain, producing a larger net welfare loss than the fixed-entry benchmark.

In our baseline, the interaction of endogenous entry and variable markups (VE–VM) compress both the intensive-margin gain and extensive-margin loss from entry and exit. Re-optimization preserves variety by keeping more firms active in the market and reduce the extensive-margin loss, but also limits domestic expansion by limiting gains at the intensive margin. Quantitatively, the smaller losses from the change in varieties outweighs the smaller gains at the intensive margin: the net welfare loss from trade war under flexible entry is mitigated by markup adjustment, as shown by the positive value of the interaction term.

To sum up, there are three main takeaways from comparing our four configurations. First, without extensive margin adjustment, trade wars have negligible effects on welfare, whether or not firms adjust markups: moving from FE–FM to FE–VM barely changes welfare for any country in our experiment, consistent with the first-order result in Section 3. Second, holding markup constant, endogenous entry and exit amplifies welfare losses substantially across all countries (see VE–FM). Third, the interaction of variable markup adjustments with entry-exit (VE–VM) mitigates these losses as re-optimization reduces the number of foreign exiters. In the baseline with production networks, the interaction is positive for every country, confirming the central prediction of Section 3.

Table 5: Welfare decomposition ($\Delta \log Y$, %)

	With Prod. Network ($\ell = 0.6$)					**Without Prod. Network** ($\ell = 1.0$)				
	(1) FE_FM	(2) FE_VM	(3) VE_FM	(4) VE_VM	**Int**	(5) FE_FM	(6) FE_VM	(7) VE_FM	(8) VE_VM	**Int**
Panel A: USA										
$\Delta \log Y$	-0.53	-0.52	-1.52	-1.26	**0.25**	-0.11	-0.10	-0.19	-0.20	**-0.02**
Intensive	-0.53	-0.52	3.44	2.19	**-1.27**	-0.11	-0.10	1.48	1.31	**-0.18**
Variety	0.00	0.00	-4.99	-3.46	**1.53**	0.00	0.00	-1.68	-1.52	**0.16**
Panel B: CHN										
$\Delta \log Y$	-0.08	-0.08	-0.20	-0.12	**0.08**	-0.06	-0.06	-0.10	-0.09	**0.01**
Intensive	-0.08	-0.08	0.33	0.28	**-0.05**	-0.06	-0.06	0.14	0.25	**0.12**
Variety	0.00	0.00	-0.54	-0.41	**0.14**	0.00	0.00	-0.23	-0.34	**-0.11**
Panel C: MEX										
$\Delta \log Y$	-0.50	-0.43	-0.83	-0.08	**0.67**	-0.11	-0.09	-0.11	-0.10	**-0.01**
Intensive	-0.50	-0.43	2.95	0.26	**-2.77**	-0.11	-0.09	-0.12	0.07	**0.17**
Variety	0.00	0.00	-3.66	-0.38	**3.28**	0.00	0.00	0.01	-0.18	**-0.18**
Panel D: CAN										
$\Delta \log Y$	-0.55	-0.45	-3.01	-0.39	**2.53**	-0.13	-0.07	0.02	-0.03	**-0.11**
Intensive	-0.55	-0.45	1.66	-0.01	**-1.76**	-0.13	-0.07	-0.30	0.00	**0.25**
Variety	0.00	0.00	-5.02	-0.38	**4.63**	0.00	0.00	0.36	-0.01	**-0.37**
Panel E: EU										
$\Delta \log Y$	0.01	0.00	-0.09	0.05	**0.14**	0.00	0.00	-0.01	0.01	**0.01**
Intensive	0.00	0.00	0.09	0.02	**-0.06**	0.00	0.00	-0.10	0.03	**0.13**
Variety	0.00	0.00	-0.17	0.03	**0.20**	0.00	0.00	0.09	-0.03	**-0.11**
Panel F: ROW										
$\Delta \log Y$	-0.01	-0.01	-0.06	0.11	**0.16**	-0.02	-0.02	-0.01	0.01	**0.02**
Intensive	-0.01	-0.01	0.05	0.00	**-0.05**	-0.02	-0.02	-0.12	0.03	**0.15**
Variety	0.00	0.00	-0.10	0.10	**0.20**	0.00	0.00	0.12	-0.02	**-0.13**

Notes: Baseline calibration. Values in percentage points. $\Delta \log Y$ = Intensive + Variety. **Int** = Interaction: VE_VM − FE_VM − VE_FM + FE_FM. Results are medians over 5 seeds.

6.4 Detailed Welfare Decomposition

In what follows, we further decompose the intensive and extensive margin adjustments into welfare relevant channels following BF decomposition developed in Section 4.

United States. Table 6 reports the results for the US. The intensive margin splits into an allocative wedge term—the welfare effect of heterogeneous markups and tariffs on prices—and an allocative factor term—the reallocation of expenditure and income across continuing firms. The allocative factor term represents how changes in factor reward affects welfare, which can be separated further into a buyer-side effect (AF buy, that measures changes in consumer price due to exposure to labor wage rate across countries through the production network) and a seller-side effect (AF sell, changes in wage bills, profits, and tariff revenue from attributed to households through operation of continuing firms). The variety margin further splits into a price component (lost or gained varieties) and an income component (lost or gained household claims from entry and exit).

First, comparing FE–FM and FE–VM columns confirms elusive results of Arkolakis, Costinot and Rodríguez-Clare (2012). Although domestic firms increase markup in response to tariff shock, which is welfare-reducing, the foreign firms lower their markups in order to stay competitive, which partly counteracts the impact of domestic firm's markup adjustment on welfare. This can be shown by a negative contribution of markup wedge from domestic firms and a positive contribution of markup wedge due to foreign firms.

Second, comparing from FE–FM to VE–FM columns reveals large welfare loss due to firm entry and exit. The intensive margin flips sign and becomes large and positive, driven by reallocation of market share: continuing domestic firms absorb the market share of exiting foreign competitors, generating large domestic wage and profit gains. The flip side is a large variety loss. On the price side, foreign firm exit destroys varieties. On the income side, the wage bills and profits attached to exiting firms vanish—almost entirely from export relationships, as US exporters discontinue their foreign operations. It turns out that the variety effect dominates the intensive-margin gains, and net welfare loss nearly triples relative to the fixed-entry benchmark.

Allowing variable markups on top of entry (VE–FM to VE–VM) compresses both margins simultaneously. On the variety side, firms that re-optimize markups remain active longer, reducing both the price-variety loss and the income lost from discontinued exports. On the intensive margin, domestic expansion diminishes: surviving foreign firms retain market share that would otherwise have been captured by domestic producers, compressing wage and profit gains attributed to US households. This reveals that the interaction between markup adjustments and firm entry-exit has opposite signs on the two margins. On the intensive side, the interaction is negative: markup adjustment mutes the domestic expansion that occurs under pure entry, because foreign firms' re-optimization preserves their presence and limits reallocation toward domestic producers. On the variety side, the interaction is positive: the variety destruction under pure entry is partially reversed when firms can adjust markups to stay in the market. These two forces partially offset, yielding a net positive interaction in total welfare.

Finally, we observe that although tariff revenue adds to the income of US households, the gain is more than offset by an increase in consumer price due to the tariff wedge, resulting in a negative direct impact of tariffs on welfare. In addition to the direct impact, tariff also induces markup adjustment and firm entry-exit, that as we demonstrated above, has overall negative effect on welfare.

Other countries. Table 7 reports the full-model (VE–VM) decomposition and interaction terms for the remaining five countries. The same economic forces operate throughout, but their relative magnitudes differ sharply with exposure to the US market.

China: limited exposure relative to economic size. Despite facing the largest bilateral tariff increase against the US, China's interaction term is small. The directly affected bilateral trade is a small fraction of its economy, so neither firm entry and exit nor markup reoptimization generates a large aggregate response. Continuing domestic firms expand only modestly, the buyer-side network channel is near zero, and variety losses remain contained.

Canada and Mexico: large interactions driven by variety preservation and input-cost relief. Canada and Mexico exhibit the largest positive interactions by far. In both countries, variable markups allow foreign firms—mainly US exporters—to stay in the market rather than exit. This preserves variety but at the same time limits domestic expansion, generating large positive interactions on variety effect but a large negative interaction on the intensive margin. The net effect is a large positive interaction in total welfare, as the reduction in variety loss outweighs the reduction in intensive-margin gain.

EU and ROW: small trade-diversion gains. The EU and ROW are less exposed to the direct tariff increases and benefit slightly from trade diversion. For these countries, the interaction is present but not central. The direct tariff shock is small, so firm entry and exit is limited; what remains is a mild combination of redirected export demand, modest variety gains, and small seller-side income gains.

Table 6: Detailed welfare decomposition: United States (%)

	(1) FE_FM	(2) FE_VM	(3) VE_FM	(4) VE_VM	**Int**
$\Delta \log Y$	-0.52	-0.51	-1.53	-1.27	**0.25**
Intensive	-0.52	-0.51	3.41	2.21	**-1.21**
Alloc wedge	-1.41	-1.51	-0.98	-1.65	**-0.57**
Markup	0.00	-0.08	0.00	-0.57	**-0.49**
dom	0.00	-0.18	0.00	-0.45	**-0.27**
for	0.00	0.10	0.00	-0.12	**-0.22**
Tariff wedge	-1.41	-1.43	-0.98	-1.08	**-0.08**
Alloc factor	0.89	1.00	4.40	3.86	**-0.64**
AF buy	0.56	0.63	2.15	2.99	**0.77**
AF sell	0.33	0.37	2.25	0.87	**-1.42**
Wage	-0.32	-0.37	1.07	-0.08	**-1.10**
dom	0.83	0.67	1.45	0.48	**-0.81**
exp	-1.16	-1.04	-0.39	-0.56	**-0.29**
Profit	-0.30	-0.25	0.53	0.22	**-0.37**
dom	0.49	0.53	0.78	0.54	**-0.28**
exp	-0.79	-0.78	-0.25	-0.32	**-0.09**
Tariff	0.96	0.99	0.65	0.74	**0.06**
Variety	0.00	0.00	-4.94	-3.48	**1.46**
Price	0.00	0.00	-1.75	-1.21	**0.54**
dom	0.00	0.00	0.00	0.00	**0.00**
for	0.00	0.00	-1.75	-1.21	**0.54**
Income	0.00	0.00	-3.19	-2.27	**0.92**
Wage	0.00	0.00	-2.62	-1.93	**0.69**
dom	0.00	0.00	0.00	0.00	**0.00**
exp	0.00	0.00	-2.62	-1.93	**0.69**
Profit	0.00	0.00	-0.51	-0.29	**0.22**
dom	0.00	0.00	0.00	0.00	**0.00**
exp	0.00	0.00	-0.51	-0.29	**0.22**
Tariff	0.00	0.00	-0.06	-0.05	**0.01**

Notes: Baseline calibration with production network ($\ell = 0.6$). Values in percentage points. $\Delta \log Y$ = Intensive + Variety. Intensive = Alloc wedge + Alloc factor. Alloc wedge = Markup (dom + for) + Tariff wedge. Alloc factor = AF_buy + AF_sell. AF_sell = Wage (dom + exp) + Profit (dom + exp) + Tariff. Variety = Price (dom + for) + Income. Income = Wage (dom + exp) + Profit (dom + exp) + Tariff. **Int** = Interaction: col. (4) − col. (2) − col. (3) + col. (1). Medians over 5 seeds.

Table 7: Detailed welfare decomposition: other countries (%)

	CHN		MEX		CAN		EU		ROW	
	VE_VM	**Int**	VE_VM	**Int**	VE_VM	**Int**	VE_VM	**Int**	VE_VM	**Int**
$\Delta \log Y$	-0.12	**0.08**	-0.11	**0.63**	-0.36	**2.69**	0.05	**0.12**	0.11	**0.16**
Intensive	0.29	**-0.08**	0.15	**-2.81**	-0.05	**-1.81**	0.02	**-0.09**	0.01	**-0.04**
Alloc wedge	-0.09	**-0.03**	-0.78	**-0.20**	-0.81	**-0.27**	-0.10	**0.00**	-0.07	**0.02**
Markup	-0.05	**-0.02**	-0.08	**-0.13**	-0.11	**-0.17**	0.00	**0.01**	0.01	**0.02**
dom	-0.05	**-0.01**	-0.05	**0.02**	-0.02	**0.02**	0.00	**0.01**	0.01	**0.03**
for	0.00	**-0.01**	-0.03	**-0.14**	-0.09	**-0.19**	0.00	**-0.01**	0.00	**-0.01**
Tariff wedge	-0.04	**-0.01**	-0.70	**-0.07**	-0.71	**-0.10**	-0.09	**-0.01**	-0.08	**0.00**
Alloc factor	0.38	**-0.05**	0.93	**-2.62**	0.76	**-1.54**	0.12	**-0.08**	0.08	**-0.05**
AF buy	0.03	**-0.41**	0.81	**3.28**	1.04	**2.36**	-0.40	**-0.16**	-0.11	**-0.41**
AF sell	0.36	**0.37**	0.12	**-5.89**	-0.28	**-3.90**	0.51	**0.08**	0.19	**0.36**
Wage	0.18	**0.20**	-0.31	**-3.87**	-0.51	**-2.63**	0.28	**0.05**	0.09	**0.21**
dom	0.18	**0.23**	0.13	**-3.46**	-0.09	**-2.68**	0.29	**0.04**	0.09	**0.23**
exp	0.00	**-0.02**	-0.44	**-0.41**	-0.42	**0.05**	-0.02	**0.01**	0.00	**-0.01**
Profit	0.16	**0.15**	-0.07	**-2.04**	-0.26	**-1.33**	0.18	**0.03**	0.05	**0.13**
dom	0.16	**0.17**	0.09	**-2.09**	-0.08	**-1.60**	0.20	**0.02**	0.05	**0.14**
exp	0.00	**-0.02**	-0.16	**0.05**	-0.18	**0.27**	-0.02	**0.00**	0.00	**-0.01**
Tariff	0.02	**0.01**	0.50	**0.02**	0.49	**0.06**	0.06	**0.01**	0.05	**0.01**
Variety	-0.42	**0.16**	-0.25	**3.45**	-0.31	**4.50**	0.03	**0.21**	0.10	**0.19**
Price	-0.14	**0.03**	-0.05	**2.07**	-0.13	**3.49**	0.01	**0.08**	0.06	**0.01**
dom	0.00	**0.00**	0.00	**0.00**	0.01	**-0.08**	0.00	**0.00**	0.00	**0.00**
for	-0.14	**0.03**	-0.05	**2.07**	-0.14	**3.57**	0.01	**0.08**	0.06	**0.01**
Income	-0.28	**0.13**	-0.20	**1.38**	-0.18	**1.02**	0.02	**0.13**	0.04	**0.19**
Wage	-0.20	**0.07**	-0.22	**1.12**	-0.19	**0.86**	0.02	**0.07**	0.03	**0.11**
dom	0.00	**0.00**	0.00	**-0.01**	0.04	**-0.26**	0.00	**0.00**	0.00	**0.00**
exp	-0.20	**0.07**	-0.22	**1.13**	-0.23	**1.11**	0.02	**0.07**	0.03	**0.11**
Profit	-0.06	**0.07**	-0.02	**0.22**	-0.02	**0.13**	0.00	**0.06**	0.00	**0.08**
dom	0.00	**0.00**	0.00	**0.00**	0.00	**-0.03**	0.00	**0.00**	0.00	**0.00**
exp	-0.06	**0.07**	-0.02	**0.23**	-0.03	**0.17**	0.00	**0.06**	0.00	**0.08**
Tariff	-0.03	**0.00**	0.04	**0.04**	0.04	**0.03**	0.00	**0.00**	0.00	**0.00**

Notes: Baseline calibration with production network ($\ell = 0.6$). Values in percentage points. $\Delta \log Y$ = Intensive + Variety. Intensive = Alloc wedge + Alloc factor. Alloc wedge = Markup (dom + for) + Tariff wedge. Alloc factor = AF_buy + AF_sell. AF_sell = Wage (dom + exp) + Profit (dom + exp) + Tariff. Variety = Price (dom + for) + Income. Income = Wage (dom + exp) + Profit (dom + exp) + Tariff. **Int** = Interaction: VE_VM − FE_VM − VE_FM + FE_FM. Medians over 5 seeds.

7 Conclusion

This paper shows that trade wars reallocate market power because they change which exporters remain active in concentrated destination markets. Using firm-level export data from 11 origins selling to 165 destinations, we document that these markets are highly granular: the median foreign product market is served by only three exporters from a given origin. In that environment, firm entry and exit is not a small extensive-margin correction. It changes the market shares, markups, and income of continuing firms, and therefore changes welfare.

The markup and entry channels of welfare have interesting interaction effects. In monopolistically competitive environments, markup adjustments have elusive aggregate welfare effects—a point made forcefully by Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2018). In the oligopolistic, granular markets we study, a complementary channel emerges: entry and exit reallocates market shares and therefore markups, and the resulting interaction is first order for welfare. Our welfare decomposition—extending Arkolakis, Costinot and Rodríguez-Clare (2012) to variable markups and tariffs and Baqaee and Farhi (2024) to discrete entry and exit—makes the markup-entry interaction measurable through a four-way counterfactual design that identifies it as a difference-in-differences.

Quantitatively, the interaction is first order. Markup adjustment alone barely moves welfare, consistent with the elusiveness result. But once entry responds, markup re-optimization preserves foreign exporters that would otherwise exit, reducing Canada's welfare loss by 87 percent (from 3.0 to 0.4 percentage points) and the US loss by 17 percent. Production networks amplify these effects sixfold, because exit in one layer raises input costs for linked producers through the supply chain.

In granular export markets, welfare assessments that hold supplier sets fixed substantially understate the cost of tariff escalation. The dominant welfare margin is not the direct reallocation of expenditure across countries, nor the pure markup adjustment that follows, but the interaction between markup changes and firm entry and exit. Policymakers evaluating trade wars—or designing tariff schedules—need models that account for firm-level market structure, endogenous participation, and the resulting reallocation of market power.

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Online Appendix for "Trade Wars and the Reallocation of Market Power in Global Export Markets"

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27 March 2026

Contents

Guide to This Online Appendix

This Online Appendix has five blocks. Appendix A documents the construction of the firm-level trade and tariff dataset and reports additional descriptive evidence on export-market concentration. Appendix B is the theory companion to Sections 3 and 4 of the main text: it collects the two-country benchmark derivations, the exact CES price algebra, and the exact ex post welfare-accounting results. Appendix C proves the exact welfare results. Appendix D keeps one approximation exercise on how decomposition error grows with tariff size. Appendix E records the numerical solution and calibration details used in the quantitative model.

Reading map for the main paper: Main-text Section 3 maps into Appendix B.1. Main-text Section 4 maps into Appendices B.2 and B.3, with proofs in Appendix C. Those appendices carry the paper's core appendix logic. Appendix D then reports one tariff-size exercise for the predictive welfare decomposition. Quantitative implementation and calibration for Sections 5 and 6 are in Appendix E, especially Subsections E.1 and E.2.

A Data Construction and Additional Evidence

This appendix documents the construction of our firm-level trade and tariff dataset and reports additional descriptive evidence on export-market concentration. The unit of observation is a firm–product–origin–destination–year shipment, and we define a *market* as an origin–destination–product–year. Using the universe of exporters for 11 low- and middle-income origins shipping to 165 destinations, we show that these markets typically contain only a handful of firms, are highly concentrated, and experience large market-share reallocation when entrants appear and incumbents exit. These facts motivate our focus on entry/exit and endogenous market power in the welfare analysis.

We construct the dataset by merging administrative customs records of firm-level exports (HS6) with product-level preferential and MFN applied tariffs from the WTO Integrated Database (WTO IDB), supplemented with Feenstra and Romalis (2014) to track phase-ins and impute missing values. Since our sample spans multiple HS revisions, we map HS6 codes into 3,646 consolidated products that are stable over time. We then construct tariff-inclusive sales values for theory-consistent market shares and use unit values as price proxies.

The customs data cover Albania, Bulgaria, Burkina Faso, Malawi, Mexico, Peru, Senegal, Uruguay, and Yemen from the World Bank Exporter Dynamics Database,[1] Egypt from the Economic Research Forum Exports Dataset, and China from the Chinese Customs Database. After cleaning and harmonization, our final estimation dataset contains 13.3 million firm–product–origin–destination–year observations spanning 2000–2013. Coverage differs by origin; 93% of observations fall in 2000–2006.

A.1 Firm-level Trade

Apart from the Chinese Customs Database, which contains monthly data for HS8 products, the raw datasets provide information on non-zero annual firm-level export values and volumes to individual foreign destinations by HS6 product. Export values are provided in US dollars and reported on a FOB basis for all countries except Senegal, which reports CIF figures. Export volumes represent net weight in kilograms, with the exception of China and Egypt, which use a variety of measures, as well as Mexico, which does not specify the measures used between 2000 and 2009. To ensure that our data are comparable across our eleven origin countries, we aggregate the monthly Chinese data to the annual level. For all eleven countries, we drop observations for which we cannot determine the destination country, observations which report a product code that

[1] The data employed in this paper are transaction-level customs data for the period 2000–2013. The data were collected by the Trade and Integration Unit of the World Bank Research Department as part of their efforts to build the Exporter Dynamics Database described in Bortoluzzi, Fernandes and Pierola (2015). The sources for the data for each country are detailed at http://econ.worldbank.org/exporter-dynamics-database.

is not part of any HS revision during our sample period and observations with missing or negative reported trade values.[2] As our dataset spans multiple revisions of the HS classification system, we further convert the raw HS6 codes to consolidated HS codes which are stable over time. The final estimation dataset contains 3646 intertemporally-consistent consolidated HS products. To create theory-consistent market share measures, we construct tariff-inclusive exports sales values by applying the relevant preferential or MFN tariff to the free-on-board export values observed in the data. Similar to other studies using administrative data, we use trade unit values as a proxy for prices.

A.2 Trade Policy

We construct bilateral ad valorem tariffs by merging the WTO Integrated Database (WTO IDB) with the phase-in information in Feenstra and Romalis (2014). The WTO IDB reports HS6-product-level preferential and applied MFN tariffs for the years 2000–2013 for 138 and 165 destination countries, respectively.[3] We map HS6 codes into our consolidated products and take a simple average across underlying HS6 lines. Following Feenstra and Romalis (2014), we impute missing MFN tariffs using the nearest available schedule (backfilling when possible, otherwise forwardfilling), and we impute missing preferential tariffs using the reported phase-in paths. For each origin–destination–product–year, we set the bilateral tariff to the lowest preferential rate available to that origin when present, and to the MFN rate otherwise.

A.3 Consolidated Product Codes

We consolidate HS codes to ensure that the product codes in our analysis are consistent over time. Our trade, tariff and commodity classification data are reported based on the HS product classification system. Since our data span a large number of years and the HS system is updated periodically, our data could feature up to four different revisions of the HS system (HS1996, HS2002, HS2007 and HS2012). We transform HS codes into consolidated HS codes which are constant over time by identifying networks of related product codes and assigning a unique consolidated code to each network, similar to Cebeci (2015). This reduces the number of distinct products in the HS system from 6,293 to 4,039. The final estimation dataset includes observations in 3646 consolidated

[2] Additionally, we drop exports from China to Hong Kong, which likely acts as an entrepot during this period.

[3] The eleven national customs databases report exports to a total of 249 foreign destinations. Omitting observations for the smaller destinations for which no tariff data is available reduces the size of the initial dataset from 26,069,241 to 24,963,950. Removing singleton observations which are absorbed by fixed effects in our baseline specification further reduces the size of our estimation dataset to 14,534,183. Dropping observations with missing destination-average tariff and origin-specific tariff due to zero trade values during 1997 - 1999 to construct the weights reduces the sample size to 13,901,014. Omitting observations with no information on control variables reduces the size of our final estimation dataset to 13,257,967.

Table A-1: Firm-level trade data: countries and years

Country	Years	Firms	Observations	... with Tariff	... with Δ Tariff
Albania	2004 - 2012	1,006	9,023	9,023	765
Bulgaria	2001 - 2006	8,922	288,945	288,945	27,297
Burkina Faso	2005 - 2007	190	1,923	1,923	71
	2008 - 2012	258	2,004	2,004	69
China	2000 - 2006	152,726	13,495,561	13,495,561	1,190,735
Egypt	2005 - 2013	7,471	246,445	246,445	23,108
Malawi	2006 - 2008	156	1,298	1,298	30
	2009 - 2012	265	2,751	2,751	151
Mexico	2000 - 2007	17,402	655,228	655,228	83,730
	2008 - 2009	9,168	202,762	202,762	7,779
	2010 - 2011	9,580	234,688	234,688	9,279
	2012	7,777	132,754	132,754	0
Peru	2000 - 2013	7,850	349,238	349,238	38,955
Senegal	2000 - 2012	840	25,183	25,183	2,559
Uruguay	2001 - 2012	1,586	60,142	60,142	8,376
Yemen	2008 - 2012	335	4,556	4,556	325

Notes: The datasets for Burkina Faso, Malawi and Mexico feature multiple distinct panels as a result of changes to the system of firm identifiers. The columns "...with Tariff" and "...with Δ Tariff" refer to the number of observations for which data on bilateral tariffs between the origin and the destination is available, and for which there is a change in the level of bilateral tariffs firms face.

Harmonized System product codes.

A.4 Stylized Facts on Market Concentration

Table A-2 summarizes export-market concentration in our data. We define a granular market as an origin–destination–product–year cell with positive trade. Panel (a) reports statistics pooling 165 destinations. Conditional on positive trade, the median number of exporters from a given origin selling a given product in a given destination is three. These markets are highly concentrated: the median top-two market share is 99.6%, and the HHI is correspondingly high. Entrants are large. Conditional on markets with at least one incumbent and one entrant (incumbents sell in both t and $t-1$, entrants sell in t but not $t-1$), the median entrant share exceeds one-third.

Panel (b) focuses on the United States. Markets are less concentrated than in the pooled sample but remain far from atomistic: the median number of exporters is seven, the median top-two share is 89.2%, and entrants capture around one-fifth of sales. These facts underscore that, even when the total number of firms selling a product in a destination may be large (see, e.g., Bernard, Eaton, Jensen and Kortum 2003; Helpman, Melitz and Yeaple 2004), the relevant competitive arena for an origin's exporters is often small. This creates scope for strategic interactions and implies that the entry or exit of a single large exporter can materially reshape market structure and pricing.

The combination of high concentration and large entrant shares implies that the entry or exit of even a single firm can substantially reshape market structure. Building on these facts, we use the same firm-level dataset to estimate the elasticities of quantities, markups, and market shares to tariff changes. A central empirical finding is that responses to origin-specific bilateral tariff changes are three to four times larger than responses to non-discriminatory tariff hikes applied to all origins. This differential response is difficult to reconcile with monopolistic competition but is consistent with oligopolistic price-setting in concentrated markets, and it provides a key motivation for the welfare analysis and quantitative model developed in the remainder of the appendix.

A.5 Additional Statistics on Market Concentration

Table A-3 reports the same concentration statistics for two additional advanced-economy destinations, the United Kingdom and Canada. The patterns mirror the pooled sample: granular origin–destination–product markets remain highly concentrated, and entrants capture meaningful shares when they enter.

Tables A-4 and A-5 break concentration down by broad HS2 industries for the full destination sample and for the United States. Concentration is high across industries in the pooled sample. The U.S. market exhibits more heterogeneity, suggesting that bilateral tariff shocks can have uneven sectoral effects on market power and welfare through differential changes in entry/exit and

Table A-2: Concentrated granular origin-destination-product markets

	25th Percentile	Median	75th Percentile
(a) All destination markets			
(i) Number of firms	1.00	3.00	7.00
(ii) Herfindahl-Hirschman Index	0.34	0.64	1.00
(iii) Top-2 market share	74.0%	99.6%	100%
(iv) Cumulative market share cond. on ≥ 1 incumbent and ≥ 1 entrant			
– Incumbents	30.3%	61.9%	85.7%
– Entrants	14.3%	38.1%	69.7%
(b) US market			
(i) Number of firms	2.00	7.00	24.00
(ii) Herfindahl-Hirschman Index	0.25	0.50	0.92
(iii) Top-2 market share	61.6%	89.2%	100%
(iv) Cumulative market share cond. on ≥ 1 incumbent and ≥ 1 entrant			
– Incumbents	49.4%	81.9%	95.2%
– Entrants	4.8%	18.1%	51.6%

Note: Markets are defined at the consolidated-HS6-product–origin–destination–year level, using an unbalanced panel of the universe of firms exporting from 11 origins to 165 destinations. Panel (a) pools all destination markets; Panel (b) restricts to the United States. Panel (iv) reports cumulative incumbent and entrant shares conditional on markets with at least one incumbent and one entrant, where incumbents sell in both t and $t-1$ and entrants sell in t but not in $t-1$.

concentration.

Table A-3: Concentrated granular origin-destination-product markets

	25th Percentile	Median	75th Percentile
(a) United Kingdom			
(i) Number of firms	1.00	3.00	10.00
(ii) Herfindahl-Hirschman Index	0.31	0.65	1.00
(iii) Top-2 market share	70.3%	99.1%	100%
(iv) Cumulative market share cond. on ≥ 1 incumbent and ≥ 1 entrant			
– Incumbents	39.9%	70.9%	89.4%
– Entrants	10.7%	29.1%	60.1%
(b) Canada			
(i) Number of firms	1.00	3.00	9.00
(ii) Herfindahl-Hirschman Index	0.33	0.66	1.00
(iii) Top-2 market share	72.6%	99.2%	100.0%
(iv) Cumulative market share cond. on ≥ 1 incumbent and ≥ 1 entrant			
– Incumbents	35.8%	68.3%	88.2%
– Entrants	11.8%	31.7%	64.2%

Note: The definitions follow Table A-2; here the destination is restricted to the United Kingdom in Panel (a) and to Canada in Panel (b).

Table A-4: Concentrated granular origin-destination-product markets - by industry

Industry (HS2 + name)	N. firms	Herfindahl index	Top-2 share	Incumbent share	Entrant share	N. markets
1-5 Live animals; animal products	2	0.69	100.00	73.79	26.21	36.8
6-14 Vegetable products	3	0.59	99.61	69.97	30.03	85.4
15 Animal/vegetable fats	2	0.84	100.00	68.72	31.28	6.3
16-24 Prepared foodstuffs	2	0.83	100.00	76.65	23.35	77.5
25-27 Mineral products	2	0.82	100.00	74.30	25.70	32.9
28-38 Products of chemical and allied industries	2	0.71	100.00	69.40	30.60	247.2
39-40 Plastics/rubber articles	2	0.73	100.00	64.89	35.11	122.4
41-43 Rawhides/leather articles, furs	3	0.61	98.42	56.39	43.61	37.7
44-46 Wood and articles of wood	3	0.64	99.89	58.89	41.11	30.6
47-49 Pulp of wood/other fibrous cellulosic materials	2	0.81	100.00	58.00	42.00	61.0
50-63 Textile and textile articles	3	0.58	97.78	53.82	46.18	370.1
64-67 Footwear, headgear, etc.	4	0.50	89.32	56.05	43.95	39.4
68-70 Misc. manufactured articles	3	0.67	99.98	59.10	40.90	100.6
71 Precious or semiprec. stones	3	0.65	99.78	63.50	36.50	13.6
72-83 Base metals and articles of base metals	3	0.65	99.54	60.81	39.19	276.1
84-85 Machinery and mechanical appliances, etc.	3	0.65	99.34	63.16	36.84	371.8
86-89 Vehicles, aircraft, etc.	3	0.59	97.99	63.60	36.40	50.0
90-92 Optical, photographic, etc.	3	0.64	99.18	60.60	39.40	100.4
93 Arms and ammunition	2	0.92	100.00	67.42	32.58	1.0
94-96 Articles of stone, plaster, etc.	4	0.53	93.42	55.70	44.30	127.0
97+ Others	2	0.80	100.00	39.43	60.57	4.1

Note: This table reports concentration statistics by broad HS2 industry for the pooled destination sample. Markets are defined as in Table A-2. The final column reports the number of origin–destination–product–year markets in each industry (in thousands). Market shares are in percent.

Table A-5: Concentrated granular origin-destination-product markets - by industry in USA

Industry (HS2 + name)	N. firms	Herfindahl index	Top-2 share	Incumbent share	Entrant share	N. markets
1-5 Live animals; animal products	5	0.51	90.96	81.51	18.49	2.0
6-14 Vegetable products	6	0.46	85.84	84.49	15.51	4.7
15 Animal/vegetable fats	4	0.66	98.46	84.84	15.16	0.5
16-24 Prepared foodstuffs	5	0.55	93.13	87.36	12.64	3.6
25-27 Mineral products	3	0.72	99.54	85.45	14.55	1.9
28-38 Products of chemical and allied industries	4	0.61	96.65	82.38	17.62	9.8
39-40 Plastics/rubber articles	8	0.51	90.72	83.56	16.44	3.6
41-43 Rawhides/leather articles, furs	12	0.36	76.68	79.55	20.45	1.3
44-46 Wood and articles of wood	12	0.36	77.41	81.57	18.43	1.3
47-49 Pulp of wood/other fibrous cellulosic materials	9	0.49	86.69	74.77	25.23	2.0
50-63 Textile and textile articles	6	0.51	90.44	73.42	26.58	17.2
64-67 Footwear, headgear, etc.	12	0.38	79.74	85.56	14.44	1.3
68-70 Misc. manufactured articles	9	0.48	86.36	84.86	15.14	3.5
71 Precious or semiprec. stones	8	0.43	82.26	80.58	19.42	0.8
72-83 Base metals and articles of base metals	7	0.52	90.66	82.97	17.03	9.7
84-85 Machinery and mechanical appliances, etc.	10	0.46	84.91	83.26	16.74	12.6
86-89 Vehicles, aircraft, etc.	6	0.55	92.33	87.90	12.10	1.9
90-92 Optical, photographic, etc.	9	0.48	85.88	83.00	17.00	4.0
93 Arms and ammunition	2	0.86	100.0	71.53	28.47	0.2
94-96 Articles of stone, plaster, etc.	15	0.4	78.75	87.08	12.92	3.4
97+ Others	7	0.44	80.93	60.36	39.64	0.4

Note: This table reports concentration statistics by broad HS2 industry for destination markets in the United States. Markets are defined as in Table A-2. The final column reports the number of origin–U.S.–product–year markets in each industry (in thousands). Market shares are in percent.

B Theory Companion: Model Derivations and Welfare Identities

This section is the theory companion to Sections 3 and 4 of the main text. It has three layers. Appendix B.1 develops the two-country benchmark and the markup–entry interaction logic. Appendix B.2 records the exact price algebra for CES aggregation, variety corrections, and network propagation. Appendix B.3 uses those exact building blocks to derive the exact ex post welfare identity and its ACR bridge. The distinction matters: the exact-hat AB system and the seven-channel results are exact once the two equilibria are observed, while Appendix D separately reports one predictive tariff-size exercise.

Status convention. In this section, "exact" means exact ex post or exact conditional on the stated active set, continuation set, or pathwise objects. Exact hat algebra gives a finite-change solution for the AB fixed-cost benchmark below, but it does not turn the full variable-markup entry problem into a global closed form.

Notation. Throughout, we index time by $t \in \{0, 1\}$ and write $\Delta \log x \equiv \log(x_1/x_0)$. In the multi-country quantitative model, we use i for product, s for sector, o for origin, d for destination, n for firm/variety, and we stack indices as subscripts when convenient (e.g., $p_{dson,t}$ is firm n's price in destination d, sector s, origin o, at time t). For any time-varying active set $\mathcal{X}_t$, we write $\mathcal{X}^C \equiv \mathcal{X}_0 \cap \mathcal{X}_1$ for the continuing set. In the two-country illustration (Appendix B.1 and the corresponding main-text discussion), we abstract from iceberg trade costs and write $\tau \geq 1$ for the gross tariff wedge applied to foreign varieties. In the multi-country quantitative model (Appendix B.2.4 onward), we follow the main-text notation in which $\tau_{iodt} \equiv 1 + t_{iodt}$ denotes the gross ad valorem tariff wedge applied in destination d to origin o (at product i and time t), while $\iota_{iodt} \geq 1$ denotes the non-tariff iceberg wedge. The notation therefore matches the main-text Section 2 discussion: in the two-country block τ is the tariff wedge itself, while in the quantitative model tariffs and non-tariff trade costs are tracked separately.

B.1 Two-Country Illustration: Exact Benchmark, Exact-Hat AB System, and Heterogeneous Firms

This subsection contains two related two-country objects. The main-text Section 3 benchmark is the Atkeson–Burstein specialization $\sigma = \rho$ with fixed-cost heterogeneity, because that is the smallest environment with exact markup-entry formulas. To show how the same mechanism changes once within-origin concentration matters separately, we start from the richer triple-nested extension with $\sigma \geq \rho > \eta$ and then note the AB specialization whenever the main-text formulas are recovered. The subsection proceeds in four blocks. First, we write the exact symmetric two-country full-GE system conditional on active firm counts in the richer extension. Second, we use that system to define the exact four-case interaction accounting. Third, we derive the fixed-entry benchmark, the exact-hat AB system used in the main text, and a short nested-demand extension with $\sigma > \rho$. Fourth, we record the finite-K heterogeneous-firm extension used in the quantitative exercises and robustness checks. The exact formulas here belong to the two-country benchmark; they are not a global closed form for the full quantitative model.

B.1.1 Setup

We start with the richer nested-demand extension. Consider two symmetric countries, Home and Foreign. Each country has J symmetric sectors. In each sector in a destination market, there are n_D domestic sellers and n_F foreign exporters. The representative consumer has triple-nested CES preferences:

$$\text{Level 1 (across sectors):} \quad C = \left(\textstyle\sum_{j=1}^{J} Q_j^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)}, \tag{B-1}$$

$$\text{Level 2 (across origins):} \quad Q_j = \left(\textstyle\sum_{o\in\{D,F\}} Q_{oj}^{(\rho-1)/\rho}\right)^{\rho/(\rho-1)}, \tag{B-2}$$

$$\text{Level 3 (across firms):} \quad Q_{oj} = \left(\textstyle\sum_{f=1}^{n_o} q_{foj}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}, \tag{B-3}$$

with elasticities satisfying $\sigma \geq \rho > \eta > 1$: firms from the same origin are more substitutable than firms from different origins, which in turn are more substitutable than goods from different sectors. Each firm produces using labor with linear technology $y = \varphi \cdot l$, with wages normalized to one. We abstract from iceberg trade costs in this block and write $\tau \geq 1$ for the gross tariff wedge applied to foreign exporters.

B.1.2 Perceived Elasticity

Under Cournot (quantity) competition, a firm internalizes that its own output affects the origin and sector aggregates in the nested CES demand system. Let $\tilde{s}_f$ denote firm f's within-origin expenditure share and let S_o denote origin o's sector expenditure share. The nested CES structure implies that the inverse demand facing firm f can be written as

$$p_f = P \cdot \left(\frac{q_f}{Q_{oj}}\right)^{-1/\sigma} \left(\frac{Q_{oj}}{Q_j}\right)^{-1/\rho} \left(\frac{Q_j}{C}\right)^{-1/\eta}, \tag{B-4}$$

where Q_{oj} is consumption of the origin-o bundle in sector j, Q_j is sector-j consumption, C is aggregate consumption, and P is the aggregate price index. Taking P and C as given from the standpoint of a single firm and differentiating yields

$$-\frac{\mathrm{d}\log p_f}{\mathrm{d}\log q_f} = \frac{1}{\sigma} + \left(\frac{1}{\rho} - \frac{1}{\sigma}\right)\tilde{s}_f + \left(\frac{1}{\eta} - \frac{1}{\rho}\right)\tilde{s}_f S_o.$$

Therefore the perceived demand elasticity is

$$\varepsilon_o = \left[\frac{1}{\sigma} + \left(\frac{1}{\rho} - \frac{1}{\sigma}\right)\tilde{s}_f + \left(\frac{1}{\eta} - \frac{1}{\rho}\right)\tilde{s}_f S_o\right]^{-1}. \tag{B-5}$$

Under symmetry within each origin, $\tilde{s}_f = 1/n_o$, so (B-5) reduces to $\varepsilon_o = [1/\sigma + (1/\rho - 1/\sigma)(1/n_o) + (1/\eta - 1/\rho)(S_o/n_o)]^{-1}$. The optimal markup is $\mu_o = \varepsilon_o/(\varepsilon_o - 1) = 1/(1 - \varepsilon_o^{-1})$.[4]

B.1.3 Markup Formulas

Substituting $\tilde{s}_D = 1/n_D$ and $\tilde{s}_F = 1/n_F$ into (B-5) yields explicit markup expressions. The domestic markup is

$$\mu_D(S_D) = \frac{1}{1 - \left[\frac{1}{\sigma} + \left(\frac{1}{\rho} - \frac{1}{\sigma}\right)\frac{1}{n_D} + \left(\frac{1}{\eta} - \frac{1}{\rho}\right)\frac{S_D}{n_D}\right]}, \tag{B-6}$$

and the foreign exporter markup is

$$\mu_F(S_D) = \frac{1}{1 - \left[\frac{1}{\sigma} + \left(\frac{1}{\rho} - \frac{1}{\sigma}\right)\frac{1}{n_F} + \left(\frac{1}{\eta} - \frac{1}{\rho}\right)\frac{1-S_D}{n_F}\right]}. \tag{B-7}$$

The markup derivatives with respect to the domestic origin share S_D are

$$\frac{d\mu_D}{dS_D} = \mu_D(S_D)^2\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\frac{1}{n_D} > 0, \qquad \frac{d\mu_F}{dS_D} = -\mu_F(S_D)^2\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\frac{1}{n_F} < 0, \tag{B-8}$$

so holding firm counts fixed, a tariff-induced increase in S_D raises domestic markups and lowers foreign exporter markups through the origin-share channel.

Reduction to Atkeson and Burstein (2008). When $\sigma = \rho$, the within-origin and across-origin elasticities coincide, and the perceived elasticity reduces to the AB Cournot formula $\varepsilon_o = [1/\sigma + (1/\eta - 1/\sigma)s_o]^{-1}$, where $s_o = S_o/n_o$ is the firm's overall sector share. As $n_D, n_F \to \infty$, individual firms become atomistic ($1/n_o \to 0$) and markups converge to the constant CES level

[4] Under Bertrand price competition with the same demand system, the perceived elasticity is $\varepsilon_o = \sigma - \tilde{s}_f[\sigma - \rho + (\rho - \eta)S_o]$.

$\sigma/(\sigma-1)$.

B.1.4 Equilibrium Characterization

The origin shares are determined by relative origin price indices. The origin-level price index for domestic goods is $P_D = n_D^{1/(1-\sigma)} \cdot p_D$ and for foreign goods is $P_F = n_F^{1/(1-\sigma)} \cdot p_F$, where $p_D = \mu_D/\varphi$ and $p_F = \mu_F \tau/\varphi$. Using the CES share identity $S_D/(1-S_D) = (P_D/P_F)^{1-\rho}$, the equilibrium domestic origin share S_D satisfies

$$\frac{S_D}{1-S_D} = \left(\frac{n_D}{n_F}\right)^{\frac{\rho-1}{\sigma-1}} \cdot \left(\frac{\mu_D(S_D)}{\mu_F(S_D)\cdot\tau}\right)^{1-\rho}. \tag{B-9}$$

Imposing $\sigma = \rho$ collapses the exponent on n_D/n_F to one and recovers the exact main-text AB share condition,

$$\frac{S_D}{1-S_D} = \left(\frac{n_D}{n_F}\right)\left(\frac{\mu_D(S_D)}{\mu_F(S_D)\tau}\right)^{1-\rho}.$$

The firm number ratio is raised to the power $(\rho-1)/(\sigma-1) < 1$ (since $\sigma > \rho$), which dampens the effect of asymmetric firm counts relative to the AB model (where the exponent is 1). When markups are variable ($\sigma > \rho$), equation (B-9) is a nonlinear equation in S_D that must be solved numerically: origin shares determine markups through both the within-origin concentration effect and the origin-share effect, and markups in turn affect origin-level price indices.

Constant-markup benchmark. When $\sigma = \rho = \eta$, markups are constant at $\mu = \sigma/(\sigma-1)$, and the equilibrium origin share has the closed-form solution

$$S_D = \frac{(n_D/n_F)\cdot\tau^{\rho-1}}{1+(n_D/n_F)\cdot\tau^{\rho-1}}. \tag{B-10}$$

B.1.5 Exact Full-GE Prices and Welfare

The origin-level price indices are

$$P_D = n_D^{1/(1-\sigma)} p_D, \qquad P_F = n_F^{1/(1-\sigma)} p_F,$$

where $p_D = \mu_D/\varphi$ and $p_F = \mu_F\tau/\varphi$. The sector price index can therefore be written as

$$P_s = P_D \cdot S_D^{1/(\rho-1)} = \frac{\mu_D}{\varphi} \cdot n_D^{\frac{1}{1-\sigma}} \cdot S_D^{\frac{1}{\rho-1}}. \tag{B-11}$$

When $\sigma = \rho$, (B-11) collapses to the exact main-text AB formula

$$P_s = \frac{\mu_D}{\varphi} \cdot n_D^{\frac{1}{1-\rho}} \cdot S_D^{\frac{1}{\rho-1}}.$$

With J symmetric sectors, the aggregate price index is $P = J^{1/(1-\eta)} \cdot P_s$.

Budget constraint. Domestic consumers receive labor income L, tariff revenue, profits from domestic sales, and profits from exports. These components satisfy

$$\text{TR} = \frac{\tau - 1}{\tau}(1 - S_D)E, \qquad \Pi_D^{\text{home}} = \left(1 - \frac{1}{\mu_D}\right) S_D E, \qquad \Pi_D^{\text{export}} = \left(1 - \frac{1}{\mu_F}\right) \frac{1 - S_D}{\tau} E. \tag{B-12}$$

Substituting (B-12) into $E = L + \text{TR} + \Pi_D^{\text{home}} + \Pi_D^{\text{export}}$ and solving yields the exact full-GE expenditure formula

$$E = \frac{L\tau}{\tau \dfrac{S_D}{\mu_D} + \dfrac{1 - S_D}{\mu_F}}. \tag{B-13}$$

Combining (B-11) and (B-13) gives the exact full-GE welfare formula

$$W = \frac{L\tau\varphi}{J^{1/(1-\eta)}\, \mu_D\, n_D^{\frac{1}{1-\sigma}}\, S_D^{\frac{1}{\rho-1}} \left(\tau \dfrac{S_D}{\mu_D} + \dfrac{1 - S_D}{\mu_F}\right)}. \tag{B-14}$$

Under the zero-profit benchmark discussed in the main text, (B-13) collapses to $E = L\tau/[1 + (\tau - 1)S_D]$ and welfare simplifies to

$$W = \frac{L\tau}{[1 + (\tau - 1)S_D] \cdot P}. \tag{B-15}$$

B.1.6 Exact Four-Case System and Welfare Decomposition

To isolate the contributions of markup adjustments and firm entry/exit, we compare welfare across four counterfactual scenarios, mirroring the definition in the two-country illustration in the main text. Let $n_{F,0}$ denote the baseline exporter count and $n_{F,1}$ the post-tariff exporter count used in the variable-entry cases. Cases 1 and 2 fix entry at $n_{F,0}$, while Cases 3 and 4 set entry at $n_{F,1}$. Cases 1 and 3 hold markups fixed at the baseline variable-markup equilibrium $(\mu_{D,0}, \mu_{F,0})$; Cases 2 and 4 let markups adjust according to (B-6)–(B-7).

For the fixed-markup cases $r \in \{1, 3\}$, domestic expenditure shares are available in closed form:

$$S_D^{(r)} = \frac{\mathcal{R}^{(r)}}{1 + \mathcal{R}^{(r)}}, \qquad \mathcal{R}^{(r)} \equiv \left(\frac{n_D}{n_F^{(r)}}\right)^{\frac{\rho-1}{\sigma-1}} \cdot \left(\frac{\mu_{D,0}}{\mu_{F,0}\tau}\right)^{1-\rho}, \tag{B-16}$$

where $n_F^{(1)} \equiv n_{F,0}$ and $n_F^{(3)} \equiv n_{F,1}$. For the variable-markup cases $r \in \{2,4\}$, domestic expenditure shares solve

$$\frac{S_D^{(r)}}{1-S_D^{(r)}} = \left(\frac{n_D}{n_F^{(r)}}\right)^{\frac{\rho-1}{\sigma-1}} \cdot \left(\frac{\mu_D(S_D^{(r)})}{\mu_F(S_D^{(r)})\cdot\tau}\right)^{1-\rho}, \tag{B-17}$$

with $n_F^{(2)} \equiv n_{F,0}$ and $n_F^{(4)} \equiv n_{F,1}$. Once $S_D^{(r)}$ is known, every regime inherits the closed-form price, expenditure, and welfare system

$$\begin{aligned} P_s^{(r)} &= \frac{\mu_D^{(r)}}{\varphi} n_D^{\frac{1}{1-\sigma}} \left(S_D^{(r)}\right)^{\frac{1}{\rho-1}}, \qquad E^{(r)} = \frac{L\tau}{\tau\dfrac{S_D^{(r)}}{\mu_D^{(r)}} + \dfrac{1-S_D^{(r)}}{\mu_F^{(r)}}}, \\ P^{(r)} &= J^{1/(1-\eta)} P_s^{(r)}, \qquad W^{(r)} = \frac{E^{(r)}}{P^{(r)}}. \end{aligned} \tag{B-18}$$

The *interaction effect* is

$$\text{Interaction} = \underbrace{\left(\Delta\log W^{(4)} - \Delta\log W^{(1)}\right)}_{\text{Total}} - \underbrace{\left(\Delta\log W^{(2)} - \Delta\log W^{(1)}\right)}_{\text{Pure markup}} - \underbrace{\left(\Delta\log W^{(3)} - \Delta\log W^{(1)}\right)}_{\text{Pure entry}}. \tag{B-19}$$

It captures the feedback loop emphasized in the main text. In the AB benchmark $\sigma = \rho$, entry already changes markups because it changes firm market shares. When $\sigma > \rho$, exporter exit also raises within-origin concentration, which further amplifies markup adjustment.

Proposition B.1 (Exact ex post: four-case interaction identity in the two-country benchmark)**.** *Define*

$$\mathcal{D}^{(r)} \equiv \tau\frac{S_D^{(r)}}{\mu_D^{(r)}} + \frac{1-S_D^{(r)}}{\mu_F^{(r)}}.$$

Then the exact interaction in log welfare can be written as

$$\begin{aligned} \mathcal{J} &\equiv \Delta\log W^{(4)} - \Delta\log W^{(3)} - \Delta\log W^{(2)} + \Delta\log W^{(1)} \\ &= -\log\left(\frac{\mathcal{D}^{(4)}\mathcal{D}^{(1)}}{\mathcal{D}^{(3)}\mathcal{D}^{(2)}}\right) - \log\left(\frac{\mu_D^{(4)}\mu_D^{(1)}}{\mu_D^{(3)}\mu_D^{(2)}}\right) \\ &\quad - \frac{1}{\rho-1}\log\left(\frac{S_D^{(4)}S_D^{(1)}}{S_D^{(3)}S_D^{(2)}}\right). \end{aligned} \tag{B-20}$$

Proof. Substitute the exact welfare formula (B-14) into the definition of $\mathcal{J}$. Constants common across the four cases cancel. Collect the remaining expenditure, markup, and share terms. □

B.1.7 Fixed-Entry Benchmark, Exact-Hat AB System, and Nested-Demand Extension

The previous subsection provided exact two-country accounting conditional on active firm counts. This subsection adds the two ingredients used in the main text. First, it records the fixed-entry benchmark and the exact-hat AB system that quantifies the markup-entry interaction for finite tariff changes. Second, it shows how the same mechanism changes once the richer nested-demand system with $\sigma > \rho$ is restored. The AB block below is exact in hats under the power-CDF fixed-cost specification. The nested-demand block is a local comparative-statics extension that isolates the additional concentration force. In logs, the four-case interaction is the difference-in-differences

$$\mathcal{J} \equiv \Delta \log W^{(4)} - \Delta \log W^{(3)} - \Delta \log W^{(2)} + \Delta \log W^{(1)} = \log\left(\frac{W^{(4)}W^{(1)}}{W^{(2)}W^{(3)}}\right).$$

Because τ is common across the four counterfactual equilibria, the interaction isolates the non-additivity between the endogenous markup response and the endogenous entry/exit response. The tariff shock matters through the pure single-margin responses M_o and N_o defined below.

Define the markup effect at fixed entry as

$$\mathcal{M} \equiv \Delta \log W^{(2)} - \Delta \log W^{(1)},$$

and the markup effect when entry is allowed as

$$\mathcal{M}^{VE} \equiv \Delta \log W^{(4)} - \Delta \log W^{(3)}.$$

By construction,

$$\mathcal{M}^{VE} = \mathcal{M} + \mathcal{J}, \tag{B-21}$$

so the interaction is exactly the wedge between markup effects with and without entry. The derivation follows the same three-step logic as the main text: first fix entry, then allow entry and exit to change markup adjustment, and finally map that loop into full-GE welfare.

A fixed-entry benchmark: why markup effects can look elusive. Let $x \equiv \log(\tau_1/\tau_0)$ and let $\lambda_D \equiv S_D$ and $\lambda_F \equiv 1 - S_D$ denote origin shares. Define the fixed-entry markup slopes

$$\beta_{\lambda,o} \equiv \left.\frac{\partial \log \mu_o}{\partial \log \lambda_o}\right|_0.$$

Holding nominal expenditure fixed and linearizing the two-origin CES block around the pre-tariff equilibrium yields

$$\mathcal{M}^{\text{price}} \equiv \Delta \log W^{(2)} - \Delta \log W^{(1)} \approx -(\rho-1)S_{D,0}(1-S_{D,0})\frac{\beta_{\lambda,D}-\beta_{\lambda,F}}{D^{10}}\,x, \tag{B-22}$$

where

$$D^{10} \equiv 1 + (\rho-1)\big[S_{D,0}\beta_{\lambda,F} + (1-S_{D,0})\beta_{\lambda,D}\big]. \tag{B-23}$$

Equation (B-22) is the ACDR-style benchmark: without entry, markup welfare depends only on the reduced-form reallocation slopes $(\beta_{\lambda,D}, \beta_{\lambda,F})$. If domestic and foreign markups move similarly at fixed entry, the welfare effect of markup adjustment is small. In the symmetric nested-demand benchmark, the fixed-entry slope is itself closed form. Holding n_o fixed in (B-5) gives

$$\beta_{\lambda,o} = \mu_{o,0}H_{o,0}S_{o,0}\left(\frac{1}{\eta}-\frac{1}{\rho}\right), \qquad H_{o,0} \equiv \frac{1}{n_{o,0}}. \tag{B-24}$$

Substituting (B-24) into (B-22) yields

$$\mathcal{M}^{\text{price}} \approx -(\rho-1)S_{D,0}(1-S_{D,0})\left(\frac{1}{\eta}-\frac{1}{\rho}\right)\frac{\mu_{D,0}H_{D,0}S_{D,0}-\mu_{F,0}H_{F,0}(1-S_{D,0})}{D^{10}}\,x, \tag{B-25}$$

with

$$D^{10} = 1 + (\rho-1)S_{D,0}(1-S_{D,0})\left(\frac{1}{\eta}-\frac{1}{\rho}\right)\left(\mu_{D,0}H_{D,0}+\mu_{F,0}H_{F,0}\right). \tag{B-26}$$

Equation (B-25) shows exactly why fixed-entry markup effects are elusive in the ACDR sense: first-order welfare depends only on the single asymmetry index $\mu_{D,0}H_{D,0}S_{D,0} - \mu_{F,0}H_{F,0}(1-S_{D,0})$. If that index is zero—in particular under a symmetric benchmark with $\mu_{D,0} = \mu_{F,0}$, $H_{D,0} = H_{F,0}$, and $S_{D,0} = 1/2$—then the first-order pure-markup welfare effect is exactly zero. The exact Section 2 benchmark confirms this directly in the symmetric free-trade calibration: the asymmetry index is numerically zero, the exact full-GE identities hold up to numerical tolerance, and the fixed-entry pure-markup effect scales as a second-order term in the tariff wedge. The underlying fixed-entry markup responses are

$$\begin{aligned}
\Delta \log \mu_D^{(2)} &\approx (\rho-1)S_{D,0}(1-S_{D,0})\frac{\mu_{D,0}H_{D,0}\left(\frac{1}{\eta}-\frac{1}{\rho}\right)}{D^{10}}\,x > 0,\\
\Delta \log \mu_F^{(2)} &\approx -(\rho-1)S_{D,0}(1-S_{D,0})\frac{\mu_{F,0}H_{F,0}\left(\frac{1}{\eta}-\frac{1}{\rho}\right)}{D^{10}}\,x < 0.
\end{aligned} \tag{B-27}$$

Thus the expanding origin's markup rises and the contracting origin's markup falls under fixed entry, but their welfare effect is first-order small around symmetric baselines because only the asymmetry term in (B-25) survives.

AB as the baseline discussion. When $\sigma = \rho$, the within-origin and across-origin nests collapse and markups depend only on firm market share $s_o \equiv S_o/n_o$, exactly as in Atkeson and Burstein (2008). Endogenous entry still matters because it changes n_o and therefore changes firm market share. In that benchmark, the markup-entry interaction already exists, but it only dampens the fixed-entry markup response. The role of $\sigma > \rho$ is narrower: it adds a separate within-origin concentration channel that can overturn the fixed-entry markup logic.

Comparison with ACDR: why entry is first order here. The contrast with Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2018) is useful because the two frameworks shut down different margins. ACDR study small changes in variable trade costs in gravity models with monopolistic competition and atomistic firms. In their baseline Pareto environment, welfare depends on one endogenous aggregate sufficient statistic: the domestic expenditure share. They also emphasize that the baseline abstracts from first-order gains from new varieties and from changes in the univariate distribution of markups. Entry and selection may still occur in the background, but they do not generate a separate first-order conduct term because each entrant is infinitesimal and incumbent pricing is not strategically linked to the number of rivals. Their fixed-marketing-cost simulations relax that benchmark, but even there the homothetic case leaves the cutoff, the markup distribution, and the number of entrants unchanged with trade costs.

Our environment differs exactly on that margin. Firms are granular and compete strategically, so the number of active suppliers is itself a conduct variable. In the AB benchmark, markups depend on firm market share $s_o = S_o/n_o$; in the richer nested-demand extension, they depend separately on the origin share S_o and within-origin concentration $H_o = 1/n_o$. Once entry or exit changes n_o, incumbent markups move even if technology and preferences are unchanged. Tariffs therefore change profits, profits change active counts, and active counts feed back into markups.

The implication is not that ACDR are overturned. Their elusiveness result is a benchmark for atomistic monopolistic competition, whereas our interaction result is a benchmark for granular oligopoly with endogenous supplier turnover. In ACDR, entry affects welfare through the domestic expenditure share and, outside the baseline, through variety terms. Here entry also changes conduct. That extra conduct channel is exactly what the markup-entry interaction $\mathcal{J}$ measures. In the AB benchmark it already makes entry first order for welfare because a change in n_o changes firm market shares discretely. When $\sigma > \rho$, the same logic is amplified by the separate within-origin concentration channel, which can even reverse the sign of the exporter markup response.

Exact-hat AB system. In the AB specialization $\sigma = \rho$, the same fixed-cost block also admits an exact finite-change representation in hats under a power-CDF family for G_o. The reason is tractability with a clear economic interpretation. The power-CDF makes the elasticity of active firm counts with respect to the profit cutoff constant, so the entire selection margin can be summarized by one parameter ζ_o. That is the smallest fixed-cost specification that keeps true endogenous entry and exit while preserving exact hat algebra. Define for example

$$G_o(f) = \left(\frac{f}{\bar{f}_o}\right)^{a_o}, \qquad a_o > 0,$$

which implies

$$n_o = M_o \left(\frac{\pi_o^{\text{var}}}{\bar{f}_o}\right)^{a_o}, \qquad \zeta_o = \frac{a_o}{1 + a_o}.$$

This makes the economic content of ζ_o transparent: it is the exact elasticity of the active-firm mass with respect to the profit shifter once the endogenous scaling by n_o is taken into account. Define baseline firm share $s_{o,0} \equiv S_{o,0}/n_{o,0}$, the markup-sensitivity coefficient

$$\xi_o \equiv \frac{\left(\frac{1}{\eta} - \frac{1}{\rho}\right) s_{o,0}}{\frac{1}{\rho} + \left(\frac{1}{\eta} - \frac{1}{\rho}\right) s_{o,0}},$$

and the baseline markup coefficient

$$\kappa_{o,0} \equiv \mu_{o,0} \left(\frac{1}{\eta} - \frac{1}{\rho}\right) s_{o,0}.$$

Then

$$\hat{z}_o = 1 + \xi_o(\hat{s}_o - 1), \qquad \hat{\mu}_o = \left[1 + \kappa_{o,0}(1 - \hat{s}_o)\right]^{-1}, \qquad \hat{s}_o \equiv \frac{\hat{S}_o}{\hat{n}_o},$$

and combining these formulas with the exact hat cutoff equations yields one exact scalar loop per origin:

$$\hat{s}_o = \hat{S}_o^{1-\zeta_o} \left(\hat{E}\hat{\tau}_o^{-1} \left[1 + \xi_o(\hat{s}_o - 1)\right]\right)^{-\zeta_o}, \qquad \hat{\tau}_D = 1, \qquad \hat{\tau}_F = \hat{\tau}.$$

This is the finite-change AB system used in main-text Propositions 1 and 2. A numerical check confirms that it matches the exact nonlinear four-case solver up to numerical tolerance over the full tariff grid used in Section 2.

Closed-form AB loop coefficients. In the AB specialization $\sigma = \rho$, markups depend only on firm market share, so (B-24) implies that the entry-to-markup slope simplifies to

$$\alpha_{\mu,o} = -\beta_{\lambda,o}.$$

Define the loop gain

$$\Gamma_o \equiv \beta_{\lambda,o}\gamma_o.$$

Then the local system solves in closed form:

$$\Delta \log \mu_o^{joint} = \frac{\beta_{\lambda,o}(1-\zeta_o)}{1+\Gamma_o}\,\Delta \log \lambda_o, \qquad \Delta \log n_o^{joint} = \frac{\zeta_o + \Gamma_o}{1+\Gamma_o}\,\Delta \log \lambda_o, \tag{B-28}$$

$$\Delta^{\text{Int}} \log \mu_o = -\frac{\beta_{\lambda,o}(\zeta_o + \Gamma_o)}{1+\Gamma_o}\,\Delta \log \lambda_o \equiv \Phi_o^{\mu}\,\Delta \log \lambda_o, \tag{B-29}$$

$$\Delta^{\text{Int}} \log n_o = \frac{\Gamma_o(1-\zeta_o)}{1+\Gamma_o}\,\Delta \log \lambda_o \equiv \Phi_o^{n}\,\Delta \log \lambda_o. \tag{B-30}$$

Under AB, the origin price index is

$$P_o = \mu_o c_o n_o^{-1/(\rho-1)}.$$

The same loop therefore implies the closed-form price interaction

$$\begin{aligned}
\Delta^{\text{Int}} \log P_o &= \left(\Phi_o^{\mu} - \frac{1}{\rho-1}\Phi_o^{n}\right)\Delta \log \lambda_o \\
&\equiv \Phi_o^{P}\,\Delta \log \lambda_o \\
&= -\frac{1}{1+\Gamma_o}\left[\beta_{\lambda,o}(\zeta_o + \Gamma_o) + \frac{\Gamma_o}{\rho-1}(1-\zeta_o)\right]\Delta \log \lambda_o.
\end{aligned} \tag{B-31}$$

These are the natural first-order summary coefficients for the AB loop: Γ_o measures feedback strength, Φ_o^n is the extra entry or exit created by allowing markups to move, Φ_o^{μ} is the resulting markup interaction, and Φ_o^P is the price interaction. The economic role of ζ_o is transparent here. It governs how much of an origin-level shock is absorbed by changes in the number of active firms rather than by larger or smaller surviving firms. The main-text propositions use the exact-hat system above because the first-order approximation is zero at exact symmetry and therefore misses finite-shock magnitudes there.

Nested-demand extension with $\sigma > \rho$. In the more general nested-demand block with $\sigma \neq \rho$, the same fixed-cost system remains tractable locally, but the coefficients no longer simplify as they do in AB because markups depend separately on origin shares and within-origin concentration.

Using (B-24), within-origin symmetry implies

$$\alpha_{\mu,o} = -\mu_{o,0} H_{o,0} \left[\left(\frac{1}{\rho} - \frac{1}{\sigma} \right) + S_{o,0} \left(\frac{1}{\eta} - \frac{1}{\rho} \right) \right], \qquad \gamma_o = \frac{\zeta_o}{\mu_{o,0} - 1}. \tag{B-32}$$

In the fixed-cost-heterogeneity block of the main text, $\beta_{n,o} = \gamma_o = \zeta_o/(\mu_{o,0} - 1)$, so the local joint markup response to an origin-share shock is

$$\left.\frac{\partial \log \mu_o}{\partial \log \lambda_o}\right|_{\text{joint}} = \frac{\mu_{o,0} H_{o,0} \left[(1-\zeta_o) S_{o,0} \left(\frac{1}{\eta} - \frac{1}{\rho} \right) - \zeta_o \left(\frac{1}{\rho} - \frac{1}{\sigma} \right) \right]}{1 - \alpha_{\mu,o}\gamma_o}. \tag{B-33}$$

Thus every coefficient in the local system can be written directly from baseline markups, firm counts, shares, and the local selection elasticity ζ_o. This is the sharp contrast with fixed entry: $\beta_{\lambda,o}$ is driven only by across-origin reallocation, whereas the joint markup slope in (B-33) also loads on the concentration term $(1/\rho - 1/\sigma)$. Under the standard ordering $\sigma > \rho > \eta$ and the local stability condition $1 - \alpha_{\mu,o}\gamma_o > 0$, the joint markup slope is negative if and only if

$$\zeta_o > \bar{\zeta}_o \equiv \frac{S_{o,0} \left(\frac{1}{\eta} - \frac{1}{\rho} \right)}{\left(\frac{1}{\rho} - \frac{1}{\sigma} \right) + S_{o,0} \left(\frac{1}{\eta} - \frac{1}{\rho} \right)}. \tag{B-34}$$

At $\sigma = \rho$, the concentration term disappears and the threshold becomes one, so the AB benchmark can dampen markup responses but cannot reverse their sign. Once $\sigma > \rho$, the separate within-origin concentration force makes sign reversal possible: domestic entry can discipline domestic markups, while foreign exit can raise the markups of surviving exporters.

Symmetry and finite shocks. At the exact symmetric benchmark used in the main text, the first-order welfare interaction still vanishes by symmetry. The relevant finite-shock object is therefore the exact nonlinear hat system, not a local derivative formula. Away from exact symmetry, the fixed-entry benchmark and the threshold (B-34) provide the useful comparative-statics summary: fixed-entry markup welfare is disciplined by one asymmetry index, while endogenous entry and exit become quantitatively important when both selection strength and markup sensitivity are large.

B.1.8 Finite-K Heterogeneous-Firm Extension

The two-country block extends directly to heterogeneous firms if each origin has a small finite set of ranked potential sellers. Let origin $o \in \{D, F\}$ have a deterministic finite-K_o productivity ladder

$$\phi_{o1} \geq \phi_{o2} \geq \cdots \geq \phi_{oK_o},$$

and let candidate active sets be top-rank sets

$$\mathcal{A}_o(0) = \varnothing, \qquad \mathcal{A}_o(n_o) = \{1, \ldots, n_o\}, \qquad n_o \in \{0, \ldots, K_o\}.$$

This preserves the discrete entry/exit logic of Section 2 while keeping the state space small enough to enumerate exactly. The deterministic step-K version is best interpreted as a tractable finite-K benchmark; the random-K version is the natural robustness check in the simulation exercises.

Conditional on an active set. Conditional on active sets $(\mathcal{A}_D, \mathcal{A}_F)$, origin and sector price indices are

$$P_o^{1-\sigma} = \sum_{k \in \mathcal{A}_o} p_{ok}^{1-\sigma}, \qquad P_s^{1-\rho} = P_D^{1-\rho} + P_F^{1-\rho}, \qquad S_o = \frac{P_o^{1-\rho}}{P_D^{1-\rho} + P_F^{1-\rho}}.$$

Firm k's within-origin and sector shares are

$$m_{ok} = \frac{p_{ok}^{1-\sigma}}{\sum_{g \in \mathcal{A}_o} p_{og}^{1-\sigma}}, \qquad s_{ok} = S_o m_{ok}.$$

The exact Cournot inverse elasticity is then

$$\frac{1}{\varepsilon_{ok}^C} = \frac{1}{\sigma} + \left(\frac{1}{\rho} - \frac{1}{\sigma}\right) m_{ok} + \left(\frac{1}{\eta} - \frac{1}{\rho}\right) s_{ok}, \qquad \mu_{ok} = \frac{1}{1 - \varepsilon_{ok}^{C,-1}}, \qquad p_{ok} = \mu_{ok} \frac{\tau_o}{\phi_{ok}},$$

where $\tau_D = 1$ and $\tau_F = \tau$. Low-productivity exit therefore raises the surviving firms' m_{ok}'s, so the same within-origin concentration force from the homogeneous-firm model survives in the heterogeneous extension. Define the within-origin Herfindahl

$$H_o \equiv \sum_{k \in \mathcal{A}_o} m_{ok}^2.$$

Exact markup aggregation. Using $1/\mu_{ok} = 1 - 1/\varepsilon_{ok}^C$, the exact profit-relevant harmonic-mean markup

$$\frac{1}{\bar{\mu}_o} \equiv \sum_{k \in \mathcal{A}_o} \frac{m_{ok}}{\mu_{ok}}$$

admits the closed form

$$\bar{\mu}_o = \left[1 - \frac{1}{\sigma} - H_o\left(\frac{1}{\rho} - \frac{1}{\sigma}\right) - S_o H_o\left(\frac{1}{\eta} - \frac{1}{\rho}\right)\right]^{-1}. \tag{B-35}$$

This is the exact heterogeneous-firm analogue of the symmetric-within-origin markup formula. The relevant concentration statistic is H_o, not $1/n_o$.

Exact finite-K benchmark with constant markups. When $\sigma = \rho = \eta$, all firms charge the same constant markup

$$\mu^* = \frac{\sigma}{\sigma - 1}.$$

Variable profits are then proportional to within-origin shares, so active sets are top-rank sets and entry is pinned down by integer zero-profit inequalities:

$$\pi^{\text{var}}_{o,n_o}(n_D, n_F) \geq f_o, \qquad \pi^{\text{var}}_{o,n_o+1}(n_o + 1, n_{-o}) < f_o.$$

This is the discrete finite-K analogue of a cutoff condition. In this benchmark, the origin share and the concentration statistic are closed form:

$$A_o(n_o) \equiv \sum_{k=1}^{n_o} \phi_{ok}^{\sigma-1}, \qquad A_o(0) \equiv 0,$$

$$S_D(n_D, n_F) = \frac{A_D(n_D)^\kappa}{A_D(n_D)^\kappa + \tau^{1-\rho} A_F(n_F)^\kappa}, \qquad H_o(n_o) = \frac{\sum_{k=1}^{n_o} \phi_{ok}^{2(\sigma-1)}}{A_o(n_o)^2}, \tag{B-36}$$

where $\kappa \equiv (\rho - 1)/(\sigma - 1)$.

Full variable-markup finite-K system. With variable markups, closed-form levels disappear, but the model remains tractable because K_D and K_F are small. For each candidate pair (n_D, n_F), one solves the firm-level fixed point for prices, shares, and markups on $\mathcal{A}_D(n_D) \times \mathcal{A}_F(n_F)$, computes the marginal active and first excluded firms' profits, and keeps the pair that satisfies the corresponding survival and no-entry inequalities. Welfare uses the exact profit-relevant harmonic-mean markups

$$\frac{1}{\bar{\mu}_o} = \sum_{k \in \mathcal{A}_o} \frac{m_{ok}}{\mu_{ok}},$$

so the two-country full-GE formulas become

$$E = \frac{L\tau}{\tau \dfrac{S_D}{\bar{\mu}_D} + \dfrac{1 - S_D}{\bar{\mu}_F}}, \qquad W = \frac{E}{P}, \qquad P = J^{1/(1-\eta)} P_s.$$

When all active firms within an origin are symmetric, $m_{ok} = 1/n_o$ and $\bar{\mu}_o = \mu_o$, so this finite-K heterogeneous block collapses to the symmetric-within-origin system above.

Relation to the main-text fixed-cost benchmark. For Section 2 in the main text, the most transparent way to introduce genuine endogenous entry and exit is to let potential firms differ only in fixed operating costs while keeping marginal costs common within an origin. Then the active firms remain symmetric conditional on (n_D, n_F), so the exact formulas in Subsections B.1.4–B.1.6 continue to hold with endogenous counts n_o. If potential firms in origin o draw fixed operating costs from a distribution $G_o(f)$ with density $g_o(f)$ and mass M_o, the exact cutoff system is

$$n_o = M_o G_o(\pi_o^{\text{var}}), \qquad \pi_D^{\text{var}} = \left(1 - \frac{1}{\mu_D}\right) \frac{S_D E}{n_D}, \qquad \pi_F^{\text{var}} = \left(1 - \frac{1}{\mu_F}\right) \frac{(1 - S_D)E}{\tau n_F}.$$

Here π_o^{var} is gross operating profit before the fixed cost is paid. It is also the cutoff object: the marginal active firm in origin o satisfies $f_o^* = \pi_o^{\text{var}}$ and therefore earns zero net profit. The local selection elasticity is

$$\chi_o \equiv \left. \frac{\pi g_o(\pi)}{G_o(\pi)} \right|_{\pi = \pi_{o,0}^{\text{var}}}, \qquad \zeta_o = \frac{\chi_o}{1 + \chi_o}, \qquad \gamma_o = \frac{\zeta_o}{\mu_{o,0} - 1}.$$

If G_o is the power-CDF family

$$G_o(f) = \left(\frac{f}{\bar{f}_o}\right)^{a_o}, \qquad a_o > 0,$$

then $\chi_o = a_o$ and $\zeta_o = a_o/(1 + a_o)$. This is why the power-CDF is useful in the main text: it turns the entire fixed-cost block into one selection parameter with a direct economic interpretation. This fixed-cost-heterogeneity version is therefore the preferred Section 2 benchmark when the priority is a closed-form characterization of markup–entry interaction. The finite-K productivity ladder above is better interpreted as the exact heterogeneous-productivity extension used for robustness checks and simulation exercises.

Numerical implementation. The exact finite-K system also has a straightforward numerical implementation. Under a flat productivity ladder it reproduces the symmetric harmonic formulas above up to numerical tolerance. Under the default step-K calibration it generates discrete active-set changes at larger tariffs and provides a finite-K benchmark for extending the Section 2 discussion beyond homogeneous firms.

B.2 Exact Price Decomposition: Sato-Vartia, Feenstra, and Networks

This subsection records the exact price algebra used later in the welfare theorem. The logic is modular. We begin with exact Sato-Vartia aggregation for CES price changes on a common comparison set, add exact Feenstra variety corrections for entry and exit, extend the result recursively through the nested CES tree, and then show how the same exact logic propagates through production networks. None of these objects is a local approximation: once the two equilibria are observed, the price decomposition is exact.

B.2.1 The Sato-Vartia Index

Consider a CES price aggregator with elasticity of substitution $\sigma > 1$ and fixed taste weights $\{\alpha_n\}_{n=1}^N$:

$$P = \left(\sum_{n=1}^{N} \alpha_n \, p_n^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{B-37}$$

For welfare decomposition, we want an exact additive representation of the finite log change $\Delta \log P$ as a weighted sum of individual log price changes $\Delta \log p_n$. The Tornqvist index uses arithmetic-mean expenditure shares $s_n^T \equiv \frac{1}{2}(s_{0,n} + s_{1,n})$, which is generally only second-order accurate: $\sum_n s_n^T \cdot \Delta \log p_n = \Delta \log P + O((\Delta \log p)^2)$. For heterogeneous price changes—such as tariff shocks affecting some goods but not others—this index-number approximation error can be non-negligible.

For CES aggregators, the Sato-Vartia index removes this approximation error. Its weights are logarithmic means.

Definition B.1 (Logarithmic Mean)**.** For $a, b > 0$, the logarithmic mean is:

$$\mathcal{L}(a, b) = \begin{cases} \dfrac{a - b}{\log a - \log b} & \text{if } a \neq b \\ a & \text{if } a = b \end{cases} \tag{B-38}$$

The logarithmic mean satisfies $\min(a, b) \leq \mathcal{L}(a, b) \leq \max(a, b)$ and lies between the geometric and arithmetic means: $\sqrt{ab} \leq \mathcal{L}(a, b) \leq \frac{a+b}{2}$.

Definition B.2 (Sato-Vartia Weights)**.** The Sato-Vartia index uses logarithmic mean weights:

$$\omega_n^{SV} = \frac{\mathcal{L}(s_{0,n}, s_{1,n})}{\sum_j \mathcal{L}(s_{0,j}, s_{1,j})} \tag{B-39}$$

where $s_{t,n} = \alpha_n p_{t,n}^{1-\sigma} / \sum_j \alpha_j p_{t,j}^{1-\sigma}$ is the expenditure share in period $t \in \{0, 1\}$.

Proposition B.2 (Sato-Vartia Exactness for CES)**.** *For any CES aggregator with elasticity σ:*

$$\sum_n \omega_n^{SV} \cdot \Delta \log p_n = \Delta \log P \quad \textit{(exactly)} \tag{B-40}$$

Proof. Let $S_t \equiv \sum_n \alpha_n p_{t,n}^{1-\sigma}$ and $s_{t,n} \equiv \alpha_n p_{t,n}^{1-\sigma}/S_t$, so $\sum_n s_{t,n} = 1$ and $\Delta \log P = \frac{1}{1-\sigma}(\log S_1 - \log S_0)$.

Since shares sum to one in each period, $\sum_n (s_{1,n} - s_{0,n}) = 0$. Applying the logarithmic-mean identity $a - b = \mathcal{L}(a,b)(\log a - \log b)$ term-by-term yields

$$0 = \sum_n \mathcal{L}(s_{0,n}, s_{1,n})\big(\log s_{1,n} - \log s_{0,n}\big).$$

Using $\log s_{t,n} = (1-\sigma)\log p_{t,n} - \log S_t$, we have $\log s_{1,n} - \log s_{0,n} = (1-\sigma)\Delta \log p_n - (\log S_1 - \log S_0)$. Substituting and rearranging gives

$$\log S_1 - \log S_0 = (1-\sigma)\frac{\sum_n \mathcal{L}(s_{0,n}, s_{1,n})\, \Delta \log p_n}{\sum_j \mathcal{L}(s_{0,j}, s_{1,j})}.$$

Dividing by $1 - \sigma$ yields $\Delta \log P = \sum_n \omega_n^{SV} \Delta \log p_n$ with ω_n^{SV} defined in (B-39). □

Table B-1 summarizes the key differences between the two indices.

Property	**Tornqvist**	**Sato-Vartia**
Weights	$\frac{1}{2}(s_{0,n} + s_{1,n})$	$\mathcal{L}(s_{0,n}, s_{1,n})/\sum_j \mathcal{L}(s_{0,j}, s_{1,j})$
Approximation order	2nd order	Exact
Additive decomposition	Yes	Yes
Nested CES	Approximate	Exact
Production networks	Approximate	Exact
Computational cost	$O(N)$	$O(N)$

Table B-1: Comparison of Tornqvist and Sato-Vartia indices for CES price aggregators.

B.2.2 Entry and Exit: The Feenstra Variety Margin

When the set of active firms changes between periods, the total price index change decomposes into intensive and extensive margins. Let $\mathcal{C}$ denote the set of *continuing* firms (active in both periods) and define:

$$\lambda_t = \frac{\sum_{n \in \mathcal{C}} \alpha_n p_{t,n}^{1-\sigma}}{\sum_{n \in \mathcal{N}_t} \alpha_n p_{t,n}^{1-\sigma}} \tag{B-41}$$

where $\mathcal{N}_t$ is the set of all active firms in period t.

Proposition B.3 (Exact Decomposition with Entry and Exit (Feenstra 1994))**.** *The total price index change decomposes exactly as:*

$$\Delta \log P = \underbrace{\sum_{n\in\mathcal{C}} \omega_n^{SV} \cdot \Delta \log p_n}_{\textit{Intensive margin } (I)} + \underbrace{\frac{1}{\sigma-1} \log \frac{\lambda_1}{\lambda_0}}_{\textit{Variety margin } (V)} \tag{B-42}$$

where the Sato-Vartia weights are computed over continuing firms only, using continuing-good shares $s_{t,n}^C \equiv \alpha_n p_{t,n}^{1-\sigma} / \sum_{j\in\mathcal{C}} \alpha_j p_{t,j}^{1-\sigma}$. This holds for any number of entrants and exiters, any pattern of price changes, and regardless of whether entry/exit is exogenous or endogenous.

Proof. Let $P_t^C \equiv \left(\sum_{n\in\mathcal{C}} \alpha_n p_{t,n}^{1-\sigma}\right)^{1/(1-\sigma)}$ denote the CES price index over the continuing set. By definition of λ_t in (B-41), $\sum_{n\in\mathcal{N}_t} \alpha_n p_{t,n}^{1-\sigma} = \left(\sum_{n\in\mathcal{C}} \alpha_n p_{t,n}^{1-\sigma}\right)/\lambda_t$, so $P_t = P_t^C \cdot \lambda_t^{1/(\sigma-1)}$. Taking log changes yields $\Delta \log P = \Delta \log P^C + \frac{1}{\sigma-1} \Delta \log \lambda$. Applying Proposition B.2 to the CES aggregator over the continuing set gives $\Delta \log P^C = \sum_{n\in\mathcal{C}} \omega_n^{SV} \Delta \log p_n$, where the Sato-Vartia weights use continuing-good shares $s_{t,n}^C$. Combining the two expressions gives (B-42). ☐

Entry ($\lambda_1 < 1$) lowers the price index through increased variety; exit ($\lambda_0 < 1$) raises it through reduced variety.

B.2.3 Nested CES Extension

Our model features a three-level nested CES structure. For each destination d, the price aggregation proceeds bottom-up:

$$P_{dso} = \left(\sum_{n\in\mathcal{N}_{dso}} p_{dson}^{1-\sigma}\right)^{1/(1-\sigma)} \qquad \text{(Firm} \rightarrow \text{Origin, elasticity } \sigma\text{)} \tag{B-43}$$

$$P_{ds} = \left(\sum_{o} P_{dso}^{1-\rho}\right)^{1/(1-\rho)} \qquad \text{(Origin} \rightarrow \text{Sector, elasticity } \rho\text{)} \tag{B-44}$$

$$P_{d} = \left(\sum_{s} P_{ds}^{1-\eta}\right)^{1/(1-\eta)} \qquad \text{(Sector} \rightarrow \text{Aggregate, elasticity } \eta\text{)} \tag{B-45}$$

To keep notation light, we suppress fixed CES taste weights (demand shifters) at each nest. All exact decompositions below are stated in terms of expenditure shares and continuing-expenditure ratios computed from the full CES system, so the presence of fixed weights is fully accounted for.

Applying the Feenstra decomposition at each nesting level yields variety terms at each level. For destination d, expanding recursively:

$$\Delta \log P_d = \underbrace{\sum_s \omega_s^{SV} \sum_o \omega_o^{SV} \sum_n \omega_n^{SV} \cdot \Delta \log p_{dson}}_{I_{\text{pure}}^{\text{nested}}}$$
$$+ \underbrace{\sum_s \omega_s^{SV} \sum_o \omega_o^{SV} \cdot V_{dso}^{\text{firm}} + \sum_s \omega_s^{SV} \cdot V_{ds}^{\text{origin}} + V_d^{\text{sector}}}_{V_d^{\text{nested}}} \tag{B-46}$$

Proposition B.4 (Nested CES Exactness)**.** *Applying Sato-Vartia weights at each CES nesting level, with Feenstra variety terms at each level, yields an exact decomposition of the aggregate price index for each destination d:*

$$\Delta \log P_d = I_{pure}^{nested} + V_d^{nested} \quad \textit{(exactly)} \tag{B-47}$$

Proof. At each CES nest, apply Proposition B.3 (with the elasticity corresponding to that nest) to decompose the log change in that nest's price index into an intensive margin over continuing units aggregated with Sato-Vartia weights and a Feenstra variety correction. The intensive margin is exact by Proposition B.2. Iterating this step through the nested structure yields (B-46) and hence the exactness claim. Proposition B.9 states the recursive exact decomposition from primitive entry/exit corrections, and Corollary B.2 gives the explicit CES formula for the three-level nest used in the main text. □

Two implementation details are critical:

Continuing units. At each CES level, Sato-Vartia weights are constructed on the *continuing set*: units with at least one sub-unit active in both periods. A unit with complete turnover (all period-0 sub-units exit and all period-1 sub-units enter) is not continuing; its effect enters through the Feenstra variety margin at that level.

True prices for shares. The CES expenditure shares used to construct Sato-Vartia weights at level ℓ are based on the true level-$(\ell - 1)$ price indices built from *all* active sub-units at that lower level, whereas the intensive-margin sum at level ℓ runs only over continuing units. This separation ensures exactness of the intensive and extensive components at every nest.

B.2.4 Channel Decomposition

In our trade model, the price of firm f from origin o selling product i to destination d satisfies:

$$\log p_{fiodt} = \log \mu_{fiodt} + \log \tau_{iodt} + \nu \cdot \log W_{ot} - \log A_{fiot} + (1-\nu) \cdot \log P_{IM,ot} \tag{B-48}$$

where μ_{fiodt} is the markup, $\tau_{iodt} \geq 1$ is the gross ad valorem tariff wedge (one plus the tariff rate) that scales purchaser prices in destination d, W_{ot} is the wage at origin o, A_{fiot} is productivity, $P_{IM,ot}$ is the intermediate-input price at origin o, and $\nu \in (0,1)$ is the labor cost share in the Cobb–Douglas technology. In the full model there is also a non-tariff iceberg wedge; in our tariff-shock exercises it is held fixed and can be absorbed into the technology term.

To keep notation readable in the decomposition formulas, we use a single micro index n to denote a continuing firm–product–origin–destination observation and suppress the underlying tuple (f, i, o, d) when summing. Objects indexed by n inherit those indices (e.g., $\Delta \log \tau_n$ is the tariff change for the corresponding (i, o, d) triple, and $\Delta \log A_n$ is the productivity change for the corresponding (f, i, o) pair).

Since log prices decompose additively, the Sato-Vartia intensive margin decomposes exactly into channels using the *same* nested weights $w_n^{\text{nested}} = \omega_n^{SV,\text{firm}} \times \omega_o^{SV,\text{origin}} \times \omega_s^{SV,\text{sector}}$:

$$\begin{aligned} I_{\text{pure}}^{\text{nested}} = & -\underbrace{\sum_n w_n^{\text{nested}} \Delta \log A_n}_{\text{tech}} + \underbrace{\sum_n w_n^{\text{nested}} \left[\Delta \log \mu_n + \Delta \log \tau_n\right]}_{\text{wedge}} \\ & + \underbrace{\sum_n w_n^{\text{nested}} \cdot \nu \cdot \Delta \log W_o}_{\text{labor}} + \underbrace{\sum_n w_n^{\text{nested}} \cdot (1-\nu) \cdot \Delta \log P_{IM,o}}_{\text{cost}_{IM}} \end{aligned} \tag{B-49}$$

This identity holds exactly because each channel uses the same Sato-Vartia weights, and the summation over continuing firms preserves the additive structure of log prices.

B.2.5 Multi-Country Leontief Propagation

The cost_{IM} term in equation (B-49) depends on equilibrium intermediate prices $\Delta \log P_{IM,o}$, which are endogenous. To eliminate this dependence, we solve the multi-country Leontief system. The full FD/IM propagation used in the seven-channel decomposition is stated in Appendix B.3.3; here we present the intermediate-price block in isolation to make the logic transparent.

Let $\beta \equiv 1 - \nu$ denote the intermediate-input cost share. Combining the channel identity with the nested Feenstra decomposition, the intermediate material price for each destination d satisfies:

$$\Delta \log P_{IM,d} = D_d + \beta \sum_o w_o^d \cdot \Delta \log P_{IM,o} + V_d^{\text{total}} \tag{B-50}$$

where D_d collects the direct (non-network) intensive-margin channels,

$$D_d \equiv \text{wedge}_d - \text{tech}_d + \text{labor}_d,$$

computed from nested Sato-Vartia weights on the continuing set,

$$\text{tech}_d \equiv \sum_n w_n^{\text{nested}} \Delta \log A_n,$$
$$\text{wedge}_d \equiv \sum_n w_n^{\text{nested}} (\Delta \log \mu_n + \Delta \log \tau_n),$$
$$\text{labor}_d \equiv \sum_n w_n^{\text{nested}} \nu \Delta \log W_o.$$

The coefficient w_o^d is the total weight of origin o in destination d's intermediate consumption (computed from continuing origins only), and V_d^{total} collects the Feenstra variety corrections across nests relevant for intermediate prices.

In matrix form, with $\Omega_{do} = \beta \cdot w_o^d$:

$$\Delta \log \boldsymbol{P}_{IM} = \boldsymbol{D} + \boldsymbol{\Omega} \cdot \Delta \log \boldsymbol{P}_{IM} + \boldsymbol{V} \tag{B-51}$$

Solving:

$$\Delta \log \boldsymbol{P}_{IM} = \underbrace{(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}}_{\boldsymbol{L}}(\boldsymbol{D} + \boldsymbol{V}) \tag{B-52}$$

The Leontief inverse $\boldsymbol{L}$ captures both direct and indirect propagation of shocks through the production network. Each element L_{do} measures destination d's total exposure to shocks originating in country o.

Proposition B.5 (Exact Propagated Decomposition)**.** *The intermediate price change for each destination d decomposes exactly into primitive shocks:*

$$-\Delta \log P_{IM,d} = \sum_o L_{do} \cdot \textit{tech}_o - \sum_o L_{do} \cdot \textit{wedge}_o - \sum_o L_{do} \cdot \textit{labor}_o - \sum_o L_{do} \cdot V_o \tag{B-53}$$

No $\textit{cost}_{IM}$ term appears in the final decomposition—it is fully absorbed into the Leontief propagation. The decomposition uses only expenditure shares (Sato-Vartia weights, continuing shares λ_t) and primitive shocks ($\Delta \log A_n$, $\Delta \log \mu_n$, $\Delta \log W_o$), not observed equilibrium prices.

Proof. From (B-52), $\Delta \log \boldsymbol{P}_{IM} = \boldsymbol{L}(\boldsymbol{D} + \boldsymbol{V})$. Taking the d-th component gives

$$\Delta \log P_{IM,d} = \sum_o L_{do}\big(D_o + V_o\big) = \sum_o L_{do}\big(\text{wedge}_o - \text{tech}_o + \text{labor}_o + V_o\big).$$

Multiplying by -1 and collecting terms yields (B-53). □

For the special case of a closed economy, $\boldsymbol{L} = \frac{1}{1-\beta}\boldsymbol{I}$, and the decomposition simplifies to $\Delta \log P = \frac{1}{1-\beta}(D + V)$, where $\frac{1}{1-\beta}$ is the standard Leontief multiplier that amplifies all shocks.

B.2.6 Two-Step Implementation for FD and IM Prices

In the full model, final demand (FD) and intermediate material (IM) sectors play distinct roles. IM prices feed back into production costs, creating the Leontief system in equation (B-52). FD prices are terminal—consumed by households, not used as inputs.

The implementation proceeds in two steps. We first build the Leontief system using IM-sector weights and solve for $\Delta \log \boldsymbol{P}_{IM}$ using equation (B-52). We then reconstruct FD price changes using the Leontief-implied IM prices in the input-cost channel:

$$\Delta \log P_{FD,d} = D_{FD,d} + \beta \sum_o w_o^{d,FD} \cdot \Delta \log P_{IM,o}^{\text{Leontief}} + V_{FD,d}^{\text{total}} \tag{B-54}$$

This two-step approach achieves reconstruction gaps below 0.005% relative to simulation-based price changes, representing near-exact precision with full channel attribution into technology, markup wedge, tariff wedge, factor income, and variety components.

B.3 Exact Seven-Channel Welfare Identity and ACR Bridge

Section 3 isolates the mechanism, but it does not deliver a global closed form for the full model. Once firm counts are discrete, markups are endogenous, entry and exit change the active set, and the economy is multi-country with production networks, one should stop looking for a single closed-form mapping from primitives to every endogenous object. This subsection does something different. Given the two equilibria, it derives exact ex post welfare accounting that keeps the main-text objects visible in a more general setting: reallocation across origins, markup wedges, variety-correction price effects, and income effects from entry and exit.

How to read this subsection. The logic proceeds in six steps. Proposition B.6 derives the sectoral origin-demand block from primitive path elasticities. Proposition B.7 turns that block into an exact nominal-anchor welfare identity. Proposition B.8, Proposition B.9, and Corollary B.2 build the exact price tree with entry and exit. Proposition B.10 and Proposition B.11 derive the exact price block, while Proposition B.12 and Corollary B.4 derive the exact income block. The two results that matter most for the main text are Proposition B.13, which combines those exact blocks into a seven-channel welfare decomposition, and Proposition B.14, which maps the ACR blocks into those same channels. In that mapping, the Section 2 mechanism appears as the exact interaction of markup reallocation, entry and exit, and the income effects associated with firm turnover.

Primitive assumptions used in this section. The exact results below rely on the following weak conditions.

1. **Welfare accounting.** Welfare satisfies $W_{d,t} = E_{d,t}/P^{FD}_{d,t}$.

2. **Primitive sectoral origin-demand system.** For each final-demand sector (d, s), the origin-demand system is differentiable along an absolutely continuous equilibrium path. Proposition B.6 uses those path elasticities to construct the finite-change objects $\mathcal{T}^{\lambda,G}_{ds}$ and $\mathcal{T}^{X}_{ds}$ rather than postulating them.

3. **Nest-level variety correction.** Whenever a nest experiences entry or exit, the counterfactual price index on the continuing set is well-defined. Under CES with fixed weights, this exact variety correction reduces to the Feenstra term.

4. **Continuing-set price propagation.** The two equilibria can be connected by an absolutely continuous equilibrium path. On the continuing producer set, each producer has a linearly homogeneous continuously differentiable unit-cost dual, and the induced propagation operator is invertible.

5. **Household price aggregation.** Household welfare prices satisfy a continuing-basket identity plus an exact variety correction, and tariffs enter purchaser prices multiplicatively when they are treated as price wedges.

6. **Exact income accounting from primitive sources.** Nominal expenditure is exactly accounted for by primitive resident-income claims on factors, firms, governments, and any other explicit transfer terms recorded in the comparison. Proposition B.12 only requires these claims to be primitive accounting objects, either because the underlying claim rules are fixed or because the claims are directly observed. Corollary B.4 then adds an arbitrary continuing/entry-exit partition used in the seven-channel statement.

These are primitive economic and regularity conditions. Exactness does *not* come from imposing ACR-style macro closures. The ACR blocks and nest-level variety corrections are endpoint objects once the two equilibria are observed. The BF-style channel terms are exact path-integral objects under the primitive theorems below; they become closed-form period-0/1 formulas only after imposing the two-basket implementation used in the quantitative model. Whenever we use the closed-form $\beta\Omega$ recursion, it is therefore an implementation corollary, not an additional welfare axiom.

B.3.1 Exact ACR-Form Identity

We write $\Delta \log x \equiv \log(x_1/x_0)$ throughout.

Proposition B.6 (Primitive sectoral origin-demand identity)**.** *Fix destination d and sector s. Let $u \in [0,1]$ index an absolutely continuous equilibrium path connecting the two equilibria. Suppose the sectoral origin-demand system is differentiable along that path and satisfies*

$$\mathrm{d}\log \lambda_{dd,s}(u) = \varepsilon^{\text{own}}_{ds}(u)\,\mathrm{d}\log\left(\frac{P_{dd,s}(u)}{P_{d,s}(u)}\right) + \sum_{o\neq d} \varepsilon^{\text{cross}}_{dso}(u)\,\mathrm{d}\log\left(\frac{P_{od,s}(u)}{P_{dd,s}(u)}\right), \tag{B-55}$$

with $\varepsilon^{\text{own}}_{ds}(u) < 0$ for all u. Then rearranging and integrating from $u = 0$ to $u = 1$ yields

$$\Delta \log P_{d,s} = \Delta \log P_{dd,s} - \mathcal{T}^{\lambda,G}_{ds} + \mathcal{T}^{X}_{ds}, \tag{B-56}$$

where

$$\mathcal{T}^{\lambda,G}_{ds} \equiv \int_0^1 \frac{1}{\varepsilon^{\text{own}}_{ds}(u)}\,\mathrm{d}\log \lambda_{dd,s}(u), \tag{B-57}$$

$$\mathcal{T}^{X}_{ds} \equiv \sum_{o\neq d} \int_0^1 \frac{\varepsilon^{\text{cross}}_{dso}(u)}{\varepsilon^{\text{own}}_{ds}(u)}\mathrm{d}\log\left(\frac{P_{od,s}(u)}{P_{dd,s}(u)}\right). \tag{B-58}$$

Proof. Rearrange (B-55) to obtain

$$\mathrm{d}\log P_{d,s}(u) = \mathrm{d}\log P_{dd,s}(u) - \frac{1}{\varepsilon^{\text{own}}_{ds}(u)}\,\mathrm{d}\log \lambda_{dd,s}(u) + \sum_{o\neq d} \frac{\varepsilon^{\text{cross}}_{dso}(u)}{\varepsilon^{\text{own}}_{ds}(u)}\mathrm{d}\log\left(\frac{P_{od,s}(u)}{P_{dd,s}(u)}\right).$$

Integrating along $u \in [0,1]$ gives (B-56) with (B-57) and (B-58). □

Proposition B.7 (Exact ex post: nominal-anchor ACR-form identity)**.** *Maintain the welfare-accounting and exact sector-aggregation assumptions stated at the start of this section, and use the sector identity from Proposition B.6. Fix destination d, let $N_d > 0$ be any nominal anchor, and* *define*

$$\mathcal{A}_d^{\lambda,G} \equiv \sum_s \omega_{ds}^{SV}\, \mathcal{T}_{ds}^{\lambda,G}, \tag{B-59}$$

$$\mathcal{A}_d^{X} \equiv \sum_s \omega_{ds}^{SV}\, \mathcal{T}_{ds}^{X}, \tag{B-60}$$

$$\mathcal{A}_d^{P} \equiv -\sum_s \omega_{ds}^{SV}\, \Delta \log\left(\frac{P_{dd,s}}{N_d}\right), \tag{B-61}$$

$$\mathcal{A}_d^{E} \equiv \Delta \log\left(\frac{E_d}{N_d}\right). \tag{B-62}$$

Then

$$\Delta \log W_d = \mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^{X} + \mathcal{A}_d^{P} + \mathcal{A}_d^{E}. \tag{B-63}$$

Equivalently, the price and income blocks are

$$\mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^{X} + \mathcal{A}_d^{P} = -\Delta \log P_d^{FD} + \Delta \log N_d, \tag{B-64}$$

$$\mathcal{A}_d^{E} = \Delta \log E_d - \Delta \log N_d. \tag{B-65}$$

Proof. By Proposition B.6,

$$\sum_s \omega_{ds}^{SV} \Delta \log P_{d,s} = \sum_s \omega_{ds}^{SV} \Delta \log P_{dd,s} \;-\; \mathcal{A}_d^{\lambda,G} \;+\; \mathcal{A}_d^{X}.$$

Exact Sato–Vartia aggregation across sectors therefore gives

$$\Delta \log P_d^{FD} = \sum_s \omega_{ds}^{SV} \Delta \log P_{dd,s} \;-\; \mathcal{A}_d^{\lambda,G} \;+\; \mathcal{A}_d^{X}.$$

Using the definition of $\mathcal{A}_d^{P}$,

$$\sum_s \omega_{ds}^{SV} \Delta \log P_{dd,s} = -\mathcal{A}_d^{P} + \Delta \log N_d,$$

which yields (B-64). Equation (B-65) is definitional. Combining the two block identities with

$$\Delta \log W_d = \Delta \log E_d - \Delta \log P_d^{FD}$$

gives (B-63). □

If the set of active final-demand sectors changes, Corollary B.2 must be applied at the sector layer before Sato–Vartia aggregation. The theorem above therefore requires exact sector aggregation on the common comparison set, not literal common activity as a primitive.

Economically, Proposition B.7 separates the same three forces that appear informally in Section 3. The term $\mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^X$ is the general reallocation block across origins. The term $\mathcal{A}_d^P$ is the domestic-price block where variable markups and entry and exit enter. The term $\mathcal{A}_d^E$ is the nominal-income block. The Section 2 markup-entry loop therefore shows up in the latter two blocks, while the origin-share movements S_o map into the first.

Corollary B.1 (Exact wage-anchor ACR-form identity under sector-specific CES across origins)**.** *Under the assumptions of Proposition B.7, assume final-demand sector activity is common across the two equilibria and CES across origins holds within each sector s with elasticity ρ_s. Choose $N_d = w_d$, let domestic labor supply L_d be fixed, and assume $1 - \tau_{TR,d,t} - \tau_{\Pi,d,t} > 0$ for $t \in \{0,1\}$. Let $\varepsilon_s \equiv 1 - \rho_s < 0$ for each sector s, and define the effective domestic price by*

$$\widehat{\widetilde{P}}_{dd}^{\,\text{eff}} \equiv \exp\Big(\sum_s \omega_{ds}^{SV}\,\Delta\log\frac{P_{dd,s}}{w_d}\Big). \tag{B-66}$$

Define income shares $\tau_{\Pi,d,t} \equiv \Pi_{d,t}/E_{d,t}$ and $\tau_{TR,d,t} \equiv TR_{d,t}/E_{d,t}$. Then

$$\widehat{W}_d = \frac{1 - \tau_{TR,d,0} - \tau_{\Pi,d,0}}{1 - \tau_{TR,d,1} - \tau_{\Pi,d,1}} \times \big(\widehat{\widetilde{P}}_{dd}^{\,\text{eff}}\big)^{-1} \times \exp\Big(\sum_s \omega_{ds}^{SV}\,\frac{1}{\varepsilon_s}\Delta\log\lambda_{dd,s}\Big). \tag{B-67}$$

In logs,

$$\begin{aligned}\Delta\log W_d = &\underbrace{\sum_s \omega_{ds}^{SV}\,\frac{1}{\varepsilon_s}\Delta\log\lambda_{dd,s}}_{\mathcal{A}_d^{\lambda}} \\ &+ \underbrace{\Delta\log\frac{1 - \tau_{TR,d,0} - \tau_{\Pi,d,0}}{1 - \tau_{TR,d,1} - \tau_{\Pi,d,1}}}_{\mathcal{A}_d^{E}} \\ &+ \underbrace{\sum_s \omega_{ds}^{SV}\,\Delta\log\Big(\frac{w_d}{P_{dd,s}}\Big)}_{\mathcal{A}_d^{P}}.\end{aligned} \tag{B-68}$$

Proof. Within each sector s, CES across origins implies

$$\Delta\log P_{d,s} = \Delta\log P_{dd,s} - \frac{1}{\varepsilon_s}\Delta\log\lambda_{dd,s},$$

so in Proposition B.7 the sectoral origin-demand block simplifies to

$$\mathcal{T}_{ds}^{\lambda,G} = \frac{1}{\varepsilon_s}\Delta\log\lambda_{dd,s}, \qquad \mathcal{T}_{ds}^{X} = 0.$$

Fixed labor supply gives

$$E_{d,t} = w_{d,t} L_d + \Pi_{d,t} + TR_{d,t} \quad \Longrightarrow \quad \frac{E_{d,t}}{w_{d,t}} = \frac{L_d}{1 - \tau_{TR,d,t} - \tau_{\Pi,d,t}}.$$

Substituting these identities into (B-63) yields (B-68). Exponentiating gives (B-67). □

B.3.2 Nested Sato-Vartia–Feenstra Price Decomposition

At each nest with entry or exit, it is useful to separate the exact variety correction from its CES closed form. The primitive object is the gap between the full price-index change and the counterfactual change computed on the continuing set alone. Under CES with fixed taste weights and a non-empty continuing set, that exact variety correction is the familiar Feenstra term. We now record that distinction explicitly and then apply the CES closed form recursively through the nesting structure. Let $\mathcal{N}_{dso,t}$ denote active firms within origin o, sector s, destination d in period t, and let $\mathcal{N}^C_{dso} \equiv \mathcal{N}_{dso,0} \cap \mathcal{N}_{dso,1}$ denote continuing firms. Let $s_{dson,t}$ denote firm n's expenditure share within the (d, s, o) nest in period t (computed using the full active set $\mathcal{N}_{dso,t}$). The continuing-expenditure ratio at the firm nest is the continuing share

$$\lambda^f_{dso,t} \equiv \sum_{n \in \mathcal{N}^C_{dso}} s_{dson,t} \in (0, 1].$$

Under CES demand with fixed taste weights, this is exactly the Feenstra continuing-expenditure ratio from Proposition B.3. Equivalently, $\lambda^f_{dso,t}$ is the expenditure share of continuing firms within the (d, s, o) nest.

Define continuing origins and sectors recursively as units with at least one continuing sub-unit: $\mathcal{O}^C_{ds} \equiv \{o : \mathcal{N}^C_{dso} \neq \emptyset\}$ and $\mathcal{S}^C_d \equiv \{s : \mathcal{O}^C_{ds} \neq \emptyset\}$. Let $s_{dso,t}$ denote origin o's expenditure share within sector (d, s) in period t (over active origins $\mathcal{O}_{ds,t}$), and let $s_{ds,t}$ denote sector s's expenditure share in destination d in period t (over active sectors $\mathcal{S}_{d,t}$). The continuing-expenditure ratios at the origin and sector nests are then

$$\lambda^o_{ds,t} \equiv \sum_{o \in \mathcal{O}^C_{ds}} s_{dso,t} \in (0, 1], \qquad \lambda^s_{d,t} \equiv \sum_{s \in \mathcal{S}^C_d} s_{ds,t} \in (0, 1],$$

where $\mathcal{O}_{ds,t}$ and $\mathcal{S}_{d,t}$ denote active origins and sectors. The same time-invariant CES-weight requirement is maintained at the origin and sector nests.

Proposition B.8 (Exact variety correction at a single nest)**.** *Fix any price aggregator P_k with a time-varying active set, and let $P^*_{k,t}$ denote the counterfactual price index obtained by evaluating the same aggregator on the continuing set only, whenever that continuing-set index is well-defined.*

Define the exact variety correction by

$$V_k^{\text{exact}} \equiv \Delta \log P_k - \Delta \log P_k^*.$$

Then

$$\Delta \log P_k = \Delta \log P_k^* + V_k^{\text{exact}}.$$

If nest k is CES with fixed weights and a non-empty continuing set, then Proposition C.1 implies

$$V_k^{\text{exact}} = \frac{1}{\theta_k} \Delta \log \lambda_k^C, \qquad \theta_k \equiv \sigma_k - 1,$$

where λ_k^C is the continuing-expenditure ratio at nest k.

Proof. The first identity is definitional. The CES closed form is exactly Feenstra's decomposition, stated below in Proposition C.1. □

Proposition B.9 (Recursive exact nested price decomposition from primitive variety corrections)**.** *Maintain Proposition B.8. Suppose the continuing-set counterfactual aggregator at each firm, origin, and sector nest has a linearly homogeneous continuously differentiable expenditure function along the comparison path. Then Shephard's lemma defines pathwise within-continuing-set expenditure shares*

$$\mathrm{d} \log P_{dso}^*(u) = \sum_{n \in \mathcal{N}_{dso}^C} \omega_{dson}^{*,f}(u) \, \mathrm{d} \log p_{dson}(u), \tag{B-69}$$

$$\mathrm{d} \log P_{ds}^*(u) = \sum_{o \in \mathcal{O}_{ds}^C} \omega_{dso}^{*,o}(u) \, \mathrm{d} \log P_{dso}(u), \tag{B-70}$$

$$\mathrm{d} \log P_{d}^*(u) = \sum_{s \in \mathcal{S}_{d}^C} \omega_{ds}^{*,s}(u) \, \mathrm{d} \log P_{ds}(u), \tag{B-71}$$

with the weights at each nest summing to one on the continuing set. Define the exact variety-correction differentials

$$\begin{aligned} \mathrm{d}V_{dso}^f(u) &\equiv \mathrm{d} \log P_{dso}(u) - \mathrm{d} \log P_{dso}^*(u), \\ \mathrm{d}V_{ds}^o(u) &\equiv \mathrm{d} \log P_{ds}(u) - \mathrm{d} \log P_{ds}^*(u), \\ \mathrm{d}V_{d}^s(u) &\equiv \mathrm{d} \log P_{d}(u) - \mathrm{d} \log P_{d}^*(u). \end{aligned}$$

Then the exact pathwise recursive decomposition is

$$\begin{aligned} \mathrm{d}\log P_d(u) = & \sum_{s\in\mathcal{S}_d^C} \omega_{ds}^{*,s}(u)\Bigg\{\sum_{o\in\mathcal{O}_{ds}^C} \omega_{dso}^{*,o}(u)\Bigg[\sum_{n\in\mathcal{N}_{dso}^C} \omega_{dson}^{*,f}(u)\,\mathrm{d}\log p_{dson}(u)\Bigg]\Bigg\} \\ & + \sum_{s\in\mathcal{S}_d^C} \omega_{ds}^{*,s}(u)\Bigg\{\sum_{o\in\mathcal{O}_{ds}^C} \omega_{dso}^{*,o}(u)\,\mathrm{d}V_{dso}^{f}(u) \;+\; \mathrm{d}V_{ds}^{o}(u)\Bigg\} \;+\; \mathrm{d}V_d^s(u). \end{aligned} \tag{B-72}$$

Integrating (B-72) *from $u = 0$ to $u = 1$ yields the exact finite-change nested decomposition. The exact nested price system is therefore constructed from primitive aggregation on the continuing set plus nest-level variety corrections. No CES restriction is needed until one wants endpoint formulas.*

Proof. By Shephard's lemma, the continuing-set expenditure function at each nest implies (B-69)–(B-71). Apply Proposition B.8 at the sector nest:

$$\mathrm{d}\log P_d(u) = \mathrm{d}\log P_d^*(u) + \mathrm{d}V_d^s(u).$$

Substitute the continuing-set representation (B-71). For each continuing sector (d, s), apply Proposition B.8 again to write

$$\mathrm{d}\log P_{ds}(u) = \mathrm{d}\log P_{ds}^*(u) + \mathrm{d}V_{ds}^o(u),$$

then substitute (B-70). Finally, for each continuing origin (d, s, o), apply Proposition B.8 once more to obtain

$$\mathrm{d}\log P_{dso}(u) = \mathrm{d}\log P_{dso}^*(u) + \mathrm{d}V_{dso}^f(u),$$

and substitute (B-69). Collecting terms yields the pathwise identity (B-72). Integrating along the absolutely continuous equilibrium path yields the finite-change statement. □

Corollary B.2 (Nested SV-Feenstra decomposition: CES specialization)**.** *Under the assumptions of Proposition B.9, further assume CES with fixed weights at each nest and non-empty continuing sets. Then the pathwise continuing-set aggregation in* (B-69)–(B-71) *integrates exactly to the corresponding Sato-Vartia formulas on the continuing set, and the variety-correction differentials integrate to the Feenstra terms*

$$V_{dso}^{f,\text{exact}} = \frac{1}{\sigma-1}\Delta\log\lambda_{dso}^f, \qquad V_{ds}^{o,\text{exact}} = \frac{1}{\rho-1}\Delta\log\lambda_{ds}^o, \qquad V_d^{s,\text{exact}} = \frac{1}{\eta-1}\Delta\log\lambda_d^s,$$

and the log change in the aggregate price index for destination d is

$$\Delta \log P_d = \sum_{s \in \mathcal{S}_d^C} \omega_{ds}^{SV} \Bigg[\sum_{o \in \mathcal{O}_{ds}^C} \omega_{dso}^{SV} \Big(\underbrace{\sum_{n \in \mathcal{N}_{dso}^C} \omega_{dson}^{SV} \, \Delta \log p_{dson}}_{I_{dso}} + \underbrace{\frac{1}{\sigma - 1} \Delta \log \lambda_{dso}^f}_{V_{dso}^f} \Big) + \underbrace{\frac{1}{\rho - 1} \Delta \log \lambda_{ds}^o}_{V_{ds}^o} \Bigg] + \underbrace{\frac{1}{\eta - 1} \Delta \log \lambda_d^s}_{V_d^s}.$$

where $\mathcal{S}_d^C$ and $\mathcal{O}_{ds}^C$ are continuing sectors and origins, all Sato-Vartia weights are computed over continuing sets, and the CES weights at each nest are time invariant. The identity holds exactly provided the continuing sets at each nest are non-empty (so the continuing-expenditure ratios are strictly positive and the log changes are well-defined).

Proof. Continuing-set intensive part. Under CES with fixed weights, Proposition B.2 gives exact Sato-Vartia aggregation on each continuing set, which is the integrated form of (B-69)–(B-71).

Variety-correction part. Under the same CES conditions, Proposition B.8 and Proposition C.1 imply

$$V_{dso}^{f,\text{exact}} = \frac{1}{\sigma - 1} \Delta \log \lambda_{dso}^f, \qquad V_{ds}^{o,\text{exact}} = \frac{1}{\rho - 1} \Delta \log \lambda_{ds}^o, \qquad V_d^{s,\text{exact}} = \frac{1}{\eta - 1} \Delta \log \lambda_d^s.$$

Substituting these continuing-set and variety-correction formulas into Proposition B.9 yields the displayed CES formula in Corollary B.2. □

B.3.3 Production-Network Propagation

The exact price-side theorem only needs primitive pathwise propagation on the continuing set. The closed-form $\beta\Omega$ recursion used in the quantitative model is a corollary of that more primitive statement, not the theorem itself.

Proposition B.10 (Exact pathwise producer-price system on the continuing set)**.** *Maintain the nest-level variety correction and continuing-set price propagation assumptions stated at the start of this section. Then for each continuing producer a,*

$$\begin{aligned} \mathrm{d} \log p_a(u) = {} & \mathrm{d} \log \mu_a(u) + \mathrm{d} \log \tau_{ad}(u) - \mathrm{d} \log A_a(u) \\ & + \sum_{j \in \mathcal{C}_a} \tilde{\Omega}_{aj}(u) \, \mathrm{d} \log p_j(u) + \sum_{f \in \mathcal{F}} \tilde{\Omega}_{af}(u) \, \mathrm{d} \log w_f(u) + \mathrm{d}V_a(u). \end{aligned} \tag{B-73}$$

Stacking across continuing producers gives

$$\big(I-\tilde{\Omega}^{NN}(u)\big)\,\mathrm{d}\log\mathbf{p}(u)=\mathrm{d}\log\boldsymbol{\mu}(u)+\mathrm{d}\log\boldsymbol{\tau}_d(u)-\mathrm{d}\log\mathbf{A}(u)+\tilde{\Omega}^{NF}(u)\,\mathrm{d}\log\mathbf{w}(u)+\mathrm{d}\mathbf{V}(u), \tag{B-74}$$

where $\mathrm{d}\mathbf{V}(u)\equiv(\mathrm{d}V_a(u))_{a\in\mathcal{N}^*}$. *Since the continuing-set propagation operator is invertible, the exact solution is*

$$\mathrm{d}\log\mathbf{p}(u)=\tilde{\Psi}(u)\Big(\mathrm{d}\log\boldsymbol{\mu}(u)+\mathrm{d}\log\boldsymbol{\tau}_d(u)-\mathrm{d}\log\mathbf{A}(u)+\mathrm{d}\mathbf{V}(u)\Big)+\tilde{\Psi}(u)\tilde{\Omega}^{NF}(u)\,\mathrm{d}\log\mathbf{w}(u), \tag{B-75}$$

with $\tilde{\Psi}(u)\equiv\big(I-\tilde{\Omega}^{NN}(u)\big)^{-1}$.

Proof. For each continuing producer a, the unit-cost dual implies

$$MC_a(u)=A_a(u)^{-1}\,c_a\big(p_{\mathcal{C}_a}(u),w(u);\mathcal{C}_a\big),$$

where $c_a(\cdot)$ is linearly homogeneous and differentiable on the continuing set. Shephard's lemma therefore gives

$$\mathrm{d}\log c_a(u)=\sum_{j\in\mathcal{C}_a}\tilde{\Omega}_{aj}(u)\,\mathrm{d}\log p_j(u)+\sum_{f\in\mathcal{F}}\tilde{\Omega}_{af}(u)\,\mathrm{d}\log w_f(u).$$

The exact variety correction at producer a's input nest adds $\mathrm{d}V_a(u)$, so

$$\mathrm{d}\log MC_a(u)=-\mathrm{d}\log A_a(u)+\sum_{j\in\mathcal{C}_a}\tilde{\Omega}_{aj}(u)\,\mathrm{d}\log p_j(u)+\sum_{f\in\mathcal{F}}\tilde{\Omega}_{af}(u)\,\mathrm{d}\log w_f(u)+\mathrm{d}V_a(u).$$

Using purchaser-price pricing, $p_a(u)=\mu_a(u)\tau_{ad}(u)MC_a(u)$, yields (B-73). Stacking the producer equations gives (B-74). Invertibility of $I-\tilde{\Omega}^{NN}(u)$ then yields (B-75). □

Proposition B.11 (Exact welfare-price decomposition under primitive pathwise conditions)**.** *Under the assumptions of Proposition B.10 and the household price-aggregation assumption stated at the start of this section, define*

$$\tilde{\lambda}^{\mathcal{C}}_{ad}(u)\equiv\sum_{j\in\mathcal{C}_d}\Omega_{dj}(u)\tilde{\Psi}_{ja}(u),\qquad\tilde{\Lambda}^{\mathcal{C}}_{fd}(u)\equiv\sum_{j\in\mathcal{C}_d}\Omega_{dj}(u)\tilde{\Psi}_{jf}(u). \tag{B-76}$$

Then destination d's welfare price index satisfies

$$\mathrm{d}\log P^{FD}_d(u)=\sum_a\tilde{\lambda}^{\mathcal{C}}_{ad}(u)\Big(\mathrm{d}\log\mu_a(u)+\mathrm{d}\log\tau_{ad}(u)-\mathrm{d}\log A_a(u)+\mathrm{d}V_a(u)\Big)+\sum_f\tilde{\Lambda}^{\mathcal{C}}_{fd}(u)\,\mathrm{d}\log w_f(u)+\mathrm{d}V_d(u). \tag{B-77}$$

Define the differential price-side channels by

$$\begin{aligned}
\mathrm{dTech}_d(u) &\equiv \sum_a \tilde{\lambda}^{\mathcal{C}}_{ad}(u)\, \mathrm{d}\log A_a(u), \\
\mathrm{dMarkupWedge}_d(u) &\equiv -\sum_a \tilde{\lambda}^{\mathcal{C}}_{ad}(u)\, \mathrm{d}\log \mu_a(u), \\
\mathrm{dTariffWedge}_d(u) &\equiv -\sum_a \tilde{\lambda}^{\mathcal{C}}_{ad}(u)\, \mathrm{d}\log \tau_{ad}(u), \\
\mathrm{dFactorBuy}_d(u) &\equiv -\sum_f \tilde{\Lambda}^{\mathcal{C}}_{fd}(u)\, \mathrm{d}\log w_f(u), \\
\mathrm{dVarietyPrice}_d(u) &\equiv -\sum_a \tilde{\lambda}^{\mathcal{C}}_{ad}(u)\, \mathrm{d}V_a(u) - \mathrm{d}V_d(u),
\end{aligned}$$

and let Tech_d, $\mathrm{MarkupWedge}_d$, $\mathrm{TariffWedge}_d$, $\mathrm{FactorBuy}_d$, *and* $\mathrm{VarietyPrice}_d$ *be the corresponding line integrals from* $u = 0$ *to* $u = 1$*. Then*

$$-\Delta \log P_d^{FD} = \mathrm{Tech}_d + \mathrm{MarkupWedge}_d + \mathrm{TariffWedge}_d + \mathrm{FactorBuy}_d + \mathrm{VarietyPrice}_d. \quad \text{(B-78)}$$

Proof. The household continuing-basket identity gives

$$\mathrm{d}\log P_d^{FD}(u) = \sum_{j \in \mathcal{C}_d} \Omega_{dj}(u)\, \mathrm{d}\log p_j(u) + \mathrm{d}V_d(u).$$

Substitute the solved producer-price system (B-75) from Proposition B.10 and rearrange sums:

$$\begin{aligned}
\sum_{j \in \mathcal{C}_d} \Omega_{dj}(u)\, \mathrm{d}\log p_j(u) = &\sum_a \tilde{\lambda}^{\mathcal{C}}_{ad}(u) \Big(\mathrm{d}\log \mu_a(u) + \mathrm{d}\log \tau_{ad}(u) - \mathrm{d}\log A_a(u) + \mathrm{d}V_a(u) \Big) \\
&+ \sum_f \tilde{\Lambda}^{\mathcal{C}}_{fd}(u)\, \mathrm{d}\log w_f(u).
\end{aligned}$$

Adding $\mathrm{d}V_d(u)$ gives (B-77). Rearranging signs defines the five differential channels, and integrating along $u \in [0, 1]$ yields (B-78). □

These five price-side channels are exact path-integral objects. Corollary B.3 provides the closed-form period-0/1 implementation used in the quantitative model.

Let $\beta \in [0, 1)$ denote the intermediate-input share. Define the IM and FD destination log price vectors by

$$\boldsymbol{p}^{IM} \equiv (\Delta \log P_d^{IM})_d, \qquad \boldsymbol{p}^{FD} \equiv (\Delta \log P_d^{FD})_d.$$

The nested SV-Feenstra decomposition delivers *direct* non-network components of these vectors. At each use, they add up into technology $\boldsymbol{T}$, markup wedge $\boldsymbol{M}$, tariff wedge $\boldsymbol{U}$, labor cost $\boldsymbol{L}$, and variety $\boldsymbol{V}$ terms. In the quantitative implementation used in the main paper, the unit-cost system has a two-basket form with common intermediate share β, so the primitive price theorem above has an explicit closed-form implementation.

Corollary B.3 (Closed-form price propagation in the two-basket implementation)**.** *Under the assumptions of Proposition B.11, further specialize to the two-basket implementation used in the quantitative model with common intermediate-input cost share $\beta \in [0,1)$. Let $\boldsymbol{\Omega}^{IM}$ be the cross-country IM origin-weight matrix and $\boldsymbol{\Omega}^{FD}$ the FD-to-origin exposure matrix. Define*

$$\boldsymbol{W}^{IM} \equiv \boldsymbol{M}^{IM} + \boldsymbol{U}^{IM}, \qquad \boldsymbol{W}^{FD} \equiv \boldsymbol{M}^{FD} + \boldsymbol{U}^{FD},$$

and the direct (non-variety) IM and FD terms

$$\boldsymbol{D}^{IM} \equiv \boldsymbol{W}^{IM} - \boldsymbol{T}^{IM} + \boldsymbol{L}^{IM}, \qquad \boldsymbol{D}^{FD} \equiv \boldsymbol{W}^{FD} - \boldsymbol{T}^{FD} + \boldsymbol{L}^{FD}.$$

If $\boldsymbol{I} - \beta\boldsymbol{\Omega}^{IM}$ is invertible, then IM prices solve

$$\boldsymbol{p}^{IM} = (\boldsymbol{I} - \beta\boldsymbol{\Omega}^{IM})^{-1}(\boldsymbol{D}^{IM} + \boldsymbol{V}^{IM}), \tag{B-79}$$

and FD prices reconstruct as

$$\boldsymbol{p}^{FD} = \boldsymbol{D}^{FD} + \beta\boldsymbol{\Omega}^{FD}\boldsymbol{p}^{IM} + \boldsymbol{V}^{FD}. \tag{B-80}$$

The negative of the FD price change equals five welfare-sign channels:

$$-\boldsymbol{p}^{FD} = \textbf{Tech} + \textbf{MarkupWedge} + \textbf{TariffWedge} + \textbf{FactorBuy} + \textbf{VarietyPrice}, \tag{B-81}$$

where the channel vectors are

$$\begin{aligned}
\textbf{Tech} &\equiv \boldsymbol{T}^{FD} + \beta\boldsymbol{\Omega}^{FD}(\boldsymbol{I} - \beta\boldsymbol{\Omega}^{IM})^{-1}\boldsymbol{T}^{IM}, \\
\textbf{MarkupWedge} &\equiv -\Big[\boldsymbol{M}^{FD} + \beta\boldsymbol{\Omega}^{FD}(\boldsymbol{I} - \beta\boldsymbol{\Omega}^{IM})^{-1}\boldsymbol{M}^{IM}\Big], \\
\textbf{TariffWedge} &\equiv -\Big[\boldsymbol{U}^{FD} + \beta\boldsymbol{\Omega}^{FD}(\boldsymbol{I} - \beta\boldsymbol{\Omega}^{IM})^{-1}\boldsymbol{U}^{IM}\Big], \\
\textbf{FactorBuy} &\equiv -\Big[\boldsymbol{L}^{FD} + \beta\boldsymbol{\Omega}^{FD}(\boldsymbol{I} - \beta\boldsymbol{\Omega}^{IM})^{-1}\boldsymbol{L}^{IM}\Big], \\
\textbf{VarietyPrice} &\equiv -\Big[\boldsymbol{V}^{FD} + \beta\boldsymbol{\Omega}^{FD}(\boldsymbol{I} - \beta\boldsymbol{\Omega}^{IM})^{-1}\boldsymbol{V}^{IM}\Big].
\end{aligned}$$

Proof. The two-basket unit-cost recursion implies

$$\boldsymbol{p}^{IM} = \boldsymbol{D}^{IM} + \beta\boldsymbol{\Omega}^{IM}\boldsymbol{p}^{IM} + \boldsymbol{V}^{IM}, \qquad \boldsymbol{p}^{FD} = \boldsymbol{D}^{FD} + \beta\boldsymbol{\Omega}^{FD}\boldsymbol{p}^{IM} + \boldsymbol{V}^{FD}.$$

Equation (B-79) solves the first system. Substitute that solution into (B-80), expand by linearity, and collect the technology, markup, tariff, labor-cost, and variety pieces. This yields (B-81) and the channel definitions. □

B.3.4 Exact Income Decomposition

To keep the income side primitive, begin with source-level resident-income accounting. Let $\mathcal{B}_d$ denote the set of nominal income sources accruing to residents of destination d: factor claims, firm claims, government rebate claims, and any other explicit transfer terms recorded in the comparison.

Proposition B.12 (Exact nominal-income accounting from primitive source claims)**.** *Maintain the exact income-accounting assumption stated at the start of this section. Suppose destination d's nominal expenditure in period t equals the sum of the income sources accruing to its residents:*

$$E_{d,t} = \sum_{b\in\mathcal{B}_d} Y_{bd,t}, \tag{B-82}$$

where $\mathcal{B}_d$ may include factor claims $Y_{fd,t} = \Phi_{df} w_{f,t} L_{f,t}$, firm claims $Y_{ad,t} = \Phi_{da}\Pi_{a,t}$, government rebate claims $Y_{\tau_d d,t} = TR_{d,t}$, and other explicit transfers. Assume only that the resident-income claims $Y_{bd,t}$ are primitive accounting objects in the comparison, for example because the underlying claim rules are fixed or because the claims are directly observed. Then

$$\Delta \log E_d = \sum_{b\in\mathcal{B}_d} \frac{\Delta Y_{bd}}{\mathcal{L}(E_{d,0}, E_{d,1})}, \tag{B-83}$$

where $\Delta Y_{bd} \equiv Y_{bd,1} - Y_{bd,0}$ and $\mathcal{L}(\cdot,\cdot)$ is the logarithmic mean. Thus the exact income side is derived from primitive source-level accounting; any continuing/entry-exit split is a regrouping of $\mathcal{B}_d$, not an additional equilibrium assumption.

Proof. Equation (B-82) gives

$$E_{d,1} - E_{d,0} = \sum_{b\in\mathcal{B}_d}(Y_{bd,1} - Y_{bd,0}) = \sum_{b\in\mathcal{B}_d} \Delta Y_{bd}.$$

Using the logarithmic-mean identity $\Delta \log E_d = (E_{d,1} - E_{d,0})/\mathcal{L}(E_{d,0}, E_{d,1})$ yields (B-83). The theorem therefore requires only exact source accounting; fixed ownership or rebate rules are one sufficient way to ensure that the source claims $Y_{bd,t}$ are primitive. □

To match the welfare channels used below, partition the primitive source set into continuing and entry/exit-related claims,

$$\mathcal{B}_d = \mathcal{B}_d^C \sqcup \mathcal{B}_d^D,$$

where $\mathcal{B}_d^C$ collects the claims treated as continuing along the comparison and $\mathcal{B}_d^D$ collects the residual claims generated by entry and exit across the two equilibria. Define

$$C_{d,t} \equiv \sum_{b \in \mathcal{B}_d^C} Y_{bd,t}, \qquad D_{d,t} \equiv \sum_{b \in \mathcal{B}_d^D} Y_{bd,t},$$

so that

$$E_{d,t} = C_{d,t} + D_{d,t},$$

with $C_{d,t} > 0$ and $D_{d,t} \geq 0$. Under the paper's baseline transfer convention, $\mathcal{B}_d^C$ consists of continuing factor, profit, and rebated tariff-revenue claims, and $\mathcal{B}_d^D$ contains the corresponding entry/exit-related claims.

Corollary B.4 (Continuing/entry-exit specialization of exact income accounting)**.** *Under the assumptions of Proposition B.12, suppose the primitive source set is partitioned as $\mathcal{B}_d = \mathcal{B}_d^C \sqcup \mathcal{B}_d^D$ with the associated objects $C_{d,t}$ and $D_{d,t}$ defined above. Define*

$$\text{FactorSell}_d \equiv \Delta \log C_d \equiv \log(C_{d,1}/C_{d,0}), \tag{B-84}$$

$$\text{VarietyIncome}_d \equiv \log \frac{1 + D_{d,1}/C_{d,1}}{1 + D_{d,0}/C_{d,0}}. \tag{B-85}$$

Then $\Delta \log E_d = \text{FactorSell}_d + \text{VarietyIncome}_d$. Under the baseline transfer convention, $\mathcal{B}_d^C$ can be regrouped into continuing wage, profit, and rebated tariff-revenue claims,

$$C_{d,t} = W_{d,t}^C + \Pi_{d,t}^C + TR_{d,t}^C,$$

in which case FactorSell_d admits the exact log-mean decomposition

$$\text{FactorSell}_d = \frac{\Delta W_d^C}{\mathcal{L}(C_{d,0}, C_{d,1})} + \frac{\Delta \Pi_d^C}{\mathcal{L}(C_{d,0}, C_{d,1})} + \frac{\Delta TR_d^C}{\mathcal{L}(C_{d,0}, C_{d,1})}, \tag{B-86}$$

where $\mathcal{L}(\cdot, \cdot)$ is the logarithmic mean.

Proof. Since $E_{d,t} = C_{d,t}(1 + D_{d,t}/C_{d,t})$, taking logs gives the first identity. Under the baseline transfer convention,

$$\Delta \log C_d = \frac{C_{d,1} - C_{d,0}}{\mathcal{L}(C_{d,0}, C_{d,1})}.$$

Using the additivity

$$C_{d,1} - C_{d,0} = \Delta W_d^C + \Delta \Pi_d^C + \Delta TR_d^C$$

yields (B-86). □

This corollary is bookkeeping, not behavior: once the primitive source set is partitioned into continuing and entry/exit-related claims, the split is an identity.

B.3.5 Seven-Channel Welfare Theorem

Combining the price-side and income-side decompositions yields the main exact result of this appendix. It answers a simple question left implicit by the ACR representation: once the two equilibria are observed, what primitive forces actually moved welfare?

Proposition B.13 (Exact ex post: seven-channel welfare decomposition)**.** *Under Proposition B.11 and Corollary B.4, for each destination d,*

$$\boxed{\begin{aligned}\Delta \log W_d = &\underbrace{\text{Tech}_d + \text{MarkupWedge}_d + \text{TariffWedge}_d}_{\textit{price wedges}} \\ &+ \underbrace{\text{FactorBuy}_d + \text{FactorSell}_d}_{\textit{income-and-input channels}} \\ &+ \underbrace{\text{VarietyPrice}_d + \text{VarietyIncome}_d}_{\textit{variety channels}}.\end{aligned}} \tag{B-87}$$

This identity is exact under the primitive conditions stated at the start of this section. In the quantitative implementation used in the main paper, Corollary B.3 supplies the closed-form period-0/1 formulas for the five price-side terms.

Relative to BF's no-entry/exit first-order accounting, the exact finite-change statement here adds two ingredients: explicit variety corrections at the affected nests and exact source-level income accounting, specialized above to the continuing/entry-exit split used in the channels.

Read Proposition B.13 as exact welfare bookkeeping. The first five channels explain why the welfare price index moved; the last two explain why nominal expenditure moved. Relative to the Section 3 mechanism, the terms to watch are MarkupWedge_d, VarietyPrice_d, VarietyIncome_d, and the profit component of FactorSell_d. MarkupWedge_d is the intensive-margin response of continuing firms' markups. VarietyPrice_d is the exact price effect of entry and exit. VarietyIncome_d and the profit part of FactorSell_d are the corresponding income effects. The remaining channels capture the direct tariff wedge and the propagation of these shocks through sectors and production networks.

Proof. Start from $\Delta \log W_d = -\Delta \log P_d^{FD} + \Delta \log E_d$. By Proposition B.11,

$$-\Delta \log P_d^{FD} = \text{Tech}_d + \text{MarkupWedge}_d + \text{TariffWedge}_d + \text{FactorBuy}_d + \text{VarietyPrice}_d.$$

By Corollary B.4,

$$\Delta \log E_d = \text{FactorSell}_d + \text{VarietyIncome}_d.$$

Summing the price and income blocks gives (B-87). □

The bridge shows how to read the same welfare change in two equivalent ways. The ACR representation isolates reallocation together with price and income corrections. The channel representation resolves those same corrections into the direct tariff wedge, the endogenous markup response of continuing firms, the price-side and income-side effects of entry and exit, and their network propagation. For the entry mechanism in Section 3, the channel representation is useful because it says which primitive force sits inside each ACR correction block.

Following BF's fictitious-factor convention, FactorSell_d is broader than labor income alone: it includes the continuing wage, profit, and tariff-revenue components collected in $C_{d,t}$.

B.3.6 ACR-to-Channel Bridge

The bridge theorem inherits the exact ACR identity together with the exact price and income decompositions proved above. Its economic content is simple: the domestic-share term still captures reallocation across origins, while the bridge theorem resolves the additional domestic-price and income corrections into primitive channels. It therefore shows exactly which forces populate each block of the extended ACR representation.

Proposition B.14 (Exact ex post: ACR-to-channel bridge)**.** *Let* $\text{AnchorNum}_d \equiv \Delta \log N_d$*. Then:*

$$\begin{aligned} \mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^X + \mathcal{A}_d^P &= \text{Tech}_d + \text{MarkupWedge}_d + \text{TariffWedge}_d \\ &\quad + \text{FactorBuy}_d + \text{VarietyPrice}_d + \text{AnchorNum}_d, \end{aligned} \tag{B-88}$$

$$\mathcal{A}_d^E = \text{FactorSell}_d + \text{VarietyIncome}_d - \text{AnchorNum}_d. \tag{B-89}$$

The nominal-anchor terms cancel exactly in total welfare:

$$\begin{aligned} \Delta \log W_d &= (\mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^X + \mathcal{A}_d^P) + \mathcal{A}_d^E \\ &= \text{Tech}_d + \text{MarkupWedge}_d + \text{TariffWedge}_d \\ &\quad + \text{FactorBuy}_d + \text{FactorSell}_d \\ &\quad + \text{VarietyPrice}_d + \text{VarietyIncome}_d. \end{aligned} \tag{B-90}$$

Proof. For the price block, Proposition B.7 gives

$$\mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^X + \mathcal{A}_d^P = -\Delta \log P_d^{FD} + \Delta \log N_d$$

Substituting the five price-side channels from Proposition B.11 gives (B-88). For the income block, Proposition B.7 gives

$$\mathcal{A}_d^E = \Delta \log E_d - \Delta \log N_d.$$

Applying Corollary B.4 gives (B-89). Adding (B-88) and (B-89), the $\pm\,\mathrm{AnchorNum}_d$ terms cancel, recovering (B-87). □

Corollary B.5 (Separating the domestic-price and reallocation ACR components)**.** *Under the assumptions of Corollary B.1, maintain the simplified wage-anchor definitions of $\mathcal{A}_d^\lambda$ and $\mathcal{A}_d^P$, and let $\varepsilon_s \equiv 1 - \rho_s < 0$ for each sector s. Then the trade-share block can be written as a relative-price object:*

$$\mathcal{A}_d^\lambda = \sum_s \omega_{ds}^{SV} \frac{1}{\varepsilon_s} \Delta \log \lambda_{dd,s} = \sum_s \omega_{ds}^{SV} \big(\Delta \log P_{dd,s} - \Delta \log P_{d,s}\big). \tag{B-91}$$

Moreover the domestic-price block expands as

$$\mathcal{A}_d^P = \sum_s \omega_{ds}^{SV} \Delta \log\left(\frac{w_d}{P_{dd,s}}\right) = -\sum_s \omega_{ds}^{SV} \Delta \log P_{dd,s} + \Delta \log w_d. \tag{B-92}$$

Finally, define sector-level price-channel components on the same continuing sets used in the seven-channel decomposition by

$$\Delta \log P_{dd,s} = \sum_{k \in \mathcal{K}_{price}} p_{k,ds}^{dd}, \qquad \Delta \log P_{d,s} = \sum_{k \in \mathcal{K}_{price}} p_{k,ds}^{agg}, \tag{B-93}$$

where each $(p_{k,ds}^{dd}, p_{k,ds}^{agg})$ is constructed using the same nested Sato–Vartia/Feenstra indexing as in the seven-channel decomposition. These are the sector-level counterparts of the aggregate price-side channels. Then each price-side channel k has a well-defined contribution to each ACR component,

$$\mathcal{A}_{k,d}^P \equiv -\sum_s \omega_{ds}^{SV} p_{k,ds}^{dd}, \qquad \mathcal{A}_{k,d}^\lambda \equiv \sum_s \omega_{ds}^{SV} \big(p_{k,ds}^{dd} - p_{k,ds}^{agg}\big), \tag{B-94}$$

so that $\mathcal{A}_d^P = \Delta \log w_d + \sum_k \mathcal{A}_{k,d}^P$ and $\mathcal{A}_d^\lambda = \sum_k \mathcal{A}_{k,d}^\lambda$. In particular, $\mathcal{A}_{k,d}^P + \mathcal{A}_{k,d}^\lambda = -\sum_s \omega_{ds}^{SV} p_{k,ds}^{agg}$, which is exactly the contribution of channel k to the aggregate price-side block $-\Delta \log P_d^{FD}$.

Proof. Equation (B-91) follows from the CES share identity

$$\Delta \log \lambda_{dd,s} = \varepsilon_s \big(\Delta \log P_{dd,s} - \Delta \log P_{d,s}\big).$$

Equation (B-92) uses $\sum_s \omega_{ds}^{SV} = 1$. Substituting (B-93) into (B-91)–(B-92) yields (B-94) and the stated aggregation identities. $\square$

C Proofs for Exact Welfare Results

C.1 Proofs of Welfare Propositions

This section proves the exact welfare-accounting results stated in Appendix B.3. It is deliberately narrower than the rest of the appendix: the two-country interaction subsection contains exact-hat algebra and local comparative statics for the benchmark mechanism, while the proofs collected here focus only on the exact ex post welfare-accounting results. Proposition B.6, Proposition B.9, Proposition B.10, and Proposition B.12 remain proved in the text because they are short primitive-to-solution derivations.

C.1.1 Proof of Proposition B.7

Proof. Step 1 (sectoral origin-demand block). By Proposition B.6,

$$\Delta \log P_{d,s} = \Delta \log P_{dd,s} - \mathcal{T}_{ds}^{\lambda,G} + \mathcal{T}_{ds}^{X}.$$

Multiplying by the Sato–Vartia weights and summing across sectors gives

$$\sum_s \omega_{ds}^{SV} \Delta \log P_{d,s} = \sum_s \omega_{ds}^{SV} \Delta \log P_{dd,s} \; - \; \mathcal{A}_d^{\lambda,G} \; + \; \mathcal{A}_d^{X}.$$

Exact sector aggregation implies

$$\Delta \log P_d^{FD} = \sum_s \omega_{ds}^{SV} \Delta \log P_{d,s},$$

so

$$\Delta \log P_d^{FD} = \sum_s \omega_{ds}^{SV} \Delta \log P_{dd,s} \; - \; \mathcal{A}_d^{\lambda,G} \; + \; \mathcal{A}_d^{X}.$$

Step 2 (price block). By definition,

$$\mathcal{A}_d^{P} = -\sum_s \omega_{ds}^{SV} \Delta \log \left(\frac{P_{dd,s}}{N_d} \right) = -\sum_s \omega_{ds}^{SV} \Delta \log P_{dd,s} + \Delta \log N_d.$$

Substituting this into the previous display gives

$$\mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^{X} + \mathcal{A}_d^{P} = -\Delta \log P_d^{FD} + \Delta \log N_d,$$

which is equation (B-64).

Step 3 (income block and welfare). Equation (B-65) is definitional:

$$\mathcal{A}_d^E = \Delta \log E_d - \Delta \log N_d.$$

Adding the two exact block identities and using

$$\Delta \log W_d = \Delta \log E_d - \Delta \log P_d^{FD}$$

yields equation (B-63). □

C.1.2 Proof of Proposition B.13

Proof. Start from the accounting identity:

$$\Delta \log W_d = -\Delta \log P_d^{FD} + \Delta \log E_d.$$

By Proposition B.11,

$$-\Delta \log P_d^{FD} = \mathrm{Tech}_d + \mathrm{MarkupWedge}_d + \mathrm{TariffWedge}_d + \mathrm{FactorBuy}_d + \mathrm{VarietyPrice}_d.$$

By exact income decomposition (Corollary B.4),

$$\Delta \log E_d = \mathrm{FactorSell}_d + \mathrm{VarietyIncome}_d.$$

Adding the two expressions gives equation (B-87). □

C.1.3 Proof of Proposition B.14

Proof. From Proposition B.7, write

$$\Delta \log W_d = \mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^X + \mathcal{A}_d^P + \mathcal{A}_d^E.$$

Using the exact price block in Proposition B.7,

$$\mathcal{A}_d^{\lambda,G} - \mathcal{A}_d^X + \mathcal{A}_d^P = -\Delta \log P_d^{FD} + \Delta \log N_d.$$

Substitute the five price-side channels from Proposition B.11 to obtain equation (B-88). Next, Proposition B.7 also gives

$$\mathcal{A}_d^E = \Delta \log E_d - \Delta \log N_d.$$

Substitute $\Delta \log E_d = \text{FactorSell}_d + \text{VarietyIncome}_d$ (Corollary B.4) to obtain equation (B-89). Summing the two block equations cancels $\pm\Delta \log N_d$ and yields equation (B-90). □

The remainder of this section provides the exact index-number and network lemmas used above.

C.1.4 Feenstra's Exact Price Index with Entry and Exit

Proposition C.1 (Exact Feenstra Variety Correction). *Fix a CES nest k with elasticity $\sigma_k > 1$ over a time-varying set of varieties $\mathcal{N}_{k,t}$, with fixed CES weights $\{\alpha_{a,k}\}_a$. Let $\mathcal{S}_k \equiv \mathcal{N}_{k,t} \cap \mathcal{N}_{k,t+1}$ denote the continuing set, and define the trade elasticity $\theta_k \equiv \sigma_k - 1$. Then the exact change in the CES price index satisfies:*

$$\frac{P_{k,t+1}}{P_{k,t}} = \frac{P^*_{k,t+1}}{P^*_{k,t}} \times \left(\frac{\lambda_{k,t+1}}{\lambda_{k,t}}\right)^{1/\theta_k} \tag{C-1}$$

where $P^*_{k,t}$ is the CES price index over continuing varieties $\mathcal{S}_k$ alone, $\lambda_{k,t} = 1 - S_x^{(k)}$ is the expenditure share of continuing varieties at time t, and $\lambda_{k,t+1} = 1 - S_e^{(k)}$ is the expenditure share of continuing varieties at time $t+1$, with $S_x^{(k)} = \sum_{a\in\mathcal{X}_k} s_{a,k,t}$ and $S_e^{(k)} = \sum_{a\in\mathcal{E}_k} s_{a,k,t+1}$ denoting the aggregate expenditure shares of exiters and entrants respectively.

Proof. The CES price index satisfies $P_{k,t}^{1-\sigma_k} = \sum_{a\in\mathcal{N}_{k,t}} \alpha_{a,k}\, p_{a,k,t}^{1-\sigma_k}$. Define the continuing-variety price index by $(P^*_{k,t})^{1-\sigma_k} = \sum_{a\in\mathcal{S}_k} \alpha_{a,k}\, p_{a,k,t}^{1-\sigma_k}$.

The expenditure share of continuing varieties is:

$$\lambda_{k,t} = \frac{\sum_{a\in\mathcal{S}_k} \alpha_{a,k}\, p_{a,k,t}^{1-\sigma_k}}{P_{k,t}^{1-\sigma_k}} = \left(\frac{P^*_{k,t}}{P_{k,t}}\right)^{1-\sigma_k},$$

so $P_{k,t} = P^*_{k,t} \cdot \lambda_{k,t}^{1/\theta_k}$. Applying the same identity in period $t+1$ and taking the ratio yields (C-1).

In log form, this gives the exact variety correction:

$$V_k^{\text{exact}} = \frac{1}{\theta_k} \log \frac{\lambda_{k,t+1}}{\lambda_{k,t}} = \frac{1}{\theta_k} \log \frac{1 - S_e^{(k)}}{1 - S_x^{(k)}}. \tag{C-2}$$

The first-order approximation $V_k^{(1)} \approx (S_x^{(k)} - S_e^{(k)})/\theta_k$ follows from $\log(1-x) \approx -x$, with approximation error of order $O\big((S^{(k)})^2\big)$. □

C.1.5 Variety Effect in the Nested Demand System

Proposition C.2 (Variety-Induced Price Change in the Nested Demand System). *In the three-level nested CES structure used in the baseline quantitative model (industries → origins → firms,* *with elasticities η, ρ, σ), the variety component of the destination-d welfare price index is, to first order:*

$$\begin{aligned}
\mathrm{d}\log P_{W_d}^{\text{Variety}} \approx\ & \underbrace{\frac{1}{\eta-1}\left(\sum_{x\in\mathcal{X}_d} s_{x|d} - \sum_{e\in\mathcal{E}_d} s_{e|d}\right)}_{\textit{industry entry/exit (typically zero)}} \\
& + \underbrace{\sum_i s_{i|d}\,\frac{1}{\rho-1}\left(\sum_{x\in\mathcal{X}_{id}} s_{x|id} - \sum_{e\in\mathcal{E}_{id}} s_{e|id}\right)}_{\textit{origin entry/exit within industry}} \\
& + \underbrace{\sum_i \sum_o s_{i|d}\, s_{o|id}\,\frac{1}{\sigma-1}\left(\sum_{x\in\mathcal{X}_{iod}} s_{x|iod} - \sum_{e\in\mathcal{E}_{iod}} s_{e|iod}\right)}_{\textit{firm entry/exit within origin}},
\end{aligned} \tag{C-3}$$

where $s_{i|d}$, $s_{o|id}$ are nested expenditure shares, and $\mathcal{X}_k$, $\mathcal{E}_k$ denote exiters and entrants at CES nest k.

Proof. Apply Proposition C.1 separately at the industry nest (d), origin nest (i,d), and firm nest (i,o,d). Then aggregate using Shephard's lemma:

$$\mathrm{d}\log P_{W_d} = \sum_i s_{i|d}\,\mathrm{d}\log p_{id}, \qquad \mathrm{d}\log p_{id} = \sum_o s_{o|id}\,\mathrm{d}\log p_{iod}.$$

Collecting only the variety (Feenstra) terms from each nest yields (C-3). □

C.1.6 Network Propagation of Variety Effects

We use the cost-based production-network notation of the main text. Let $\mathcal{N}^*$ denote the set of continuing producers and $\mathcal{F}$ the set of primary factors. Let Ω_{aj} denote revenue-based input shares and define cost-based shares $\tilde{\Omega}_{aj} \equiv \mu_a\,\Omega_{aj}$, where μ_a is the gross markup of producer a. Let $\tilde{\Omega}^{NN}$ restrict the cost-based input-share matrix to producers in $\mathcal{N}^*$ and assume the corresponding cost-based Leontief inverse exists:

$$\tilde{\Psi} \equiv (I - \tilde{\Omega}^{NN})^{-1}.$$

For destination d, let $\mathcal{C}_d$ denote the continuing set in d's welfare basket and let Ω_{dj} denote d's expenditure share on continuing variety $j \in \mathcal{C}_d$. Define the continuing-basket cost-based exposures

$$\tilde{\lambda}^{\mathcal{C}}_{ad} \equiv \sum_{j\in\mathcal{C}_d} \Omega_{dj}\,\tilde{\Psi}_{ja}, \qquad \tilde{\Lambda}^{\mathcal{C}}_{fd} \equiv \sum_{j\in\mathcal{C}_d} \Omega_{dj}\,\tilde{\Psi}_{jf},$$

and the welfare-relevant continuing producer set $\mathcal{C}_d^{\mathrm{W}} \equiv \{a \in \mathcal{N}^* : \tilde{\lambda}_{ad}^{\mathcal{C}} > 0\}$. The next result is a no-entry/exit specialization of Propositions B.10 and B.11, recorded because it is the convenient implementation object used in the operational first-order corollary below.

Proposition C.3 (Cost-based price propagation on the continuing set (no-entry/exit corollary))**.** *On the continuing set, prices satisfy the log-linear system*

$$\mathrm{d}\log p_a = \mathrm{d}\log \mu_a + \mathrm{d}\log \tau_{ad} - \mathrm{d}\log A_a + \sum_{j\in\mathcal{N}^*} \tilde{\Omega}_{aj}\,\mathrm{d}\log p_j + \sum_{f\in\mathcal{F}} \tilde{\Omega}_{af}\,\mathrm{d}\log w_f, \qquad a \in \mathcal{N}^*,$$

and the induced change in destination d's welfare deflator restricted to continuing varieties is

$$\mathrm{d}\log P_{W_d}^{\mathrm{Cont.}} = \sum_{a\in\mathcal{C}_d^{\mathrm{W}}} \tilde{\lambda}_{ad}^{\mathcal{C}}\big(\mathrm{d}\log \mu_a + \mathrm{d}\log \tau_{ad} - \mathrm{d}\log A_a\big) + \sum_{f\in\mathcal{F}} \tilde{\Lambda}_{fd}^{\mathcal{C}}\,\mathrm{d}\log w_f. \tag{C-4}$$

Proof. Suppress the path index u and specialize Proposition B.10 to the no-entry/exit case with $\mathrm{d}V_a = 0$ for all continuing producers. This gives the displayed producer-price system. Then specialize Proposition B.11 to $\mathrm{d}V_d = 0$ and the same no-entry/exit environment. The continuing welfare-basket price change becomes

$$\mathrm{d}\log P_{W_d}^{\mathrm{Cont.}} = \sum_{j\in\mathcal{C}_d} \Omega_{dj}\,\mathrm{d}\log p_j,$$

and substituting the solved producer-price system, then collecting coefficients on producer and factor price movements, yields (C-4). □

In a production network, variety shocks can also occur at intermediate-input nests, not just in final demand. Let $\mathcal{K}^d$ denote the set of CES nests affecting destination d's welfare: $\mathcal{K}^d = \{d\}\cup\{a \in \mathcal{N}^* : \tilde{\lambda}_{ad}^{\mathcal{C}} > 0\}$.

Proposition C.4 (Network Variety Effect)**.** *For each continuing producer $a \in \mathcal{N}^*$, the variety shock V_a at a's input nest enters the cost system as:*

$$\mathrm{d}\log P_a^{\mathrm{input}} = \sum_{j\in\mathcal{N}^*} \tilde{\Omega}_{aj}\,\mathrm{d}\log p_j + \sum_{f} \tilde{\Omega}_{af}\,\mathrm{d}\log w_f + V_a.$$

Solving with the cost-based Leontief inverse $\tilde{\Psi} = (I - \tilde{\Omega}^{NN})^{-1}$, the variety component of the welfare price index is:

$$\mathrm{d}\log P_{W_d}^{\mathrm{Variety}} = V_d + \sum_{a\in\mathcal{N}^*} \tilde{\lambda}_{ad}^{\mathcal{C}}\,V_a, \tag{C-5}$$

and the implied network variety effect on welfare is:

$$E_d^{\text{network}} = -\mathrm{d}\log P_{W_d}^{\text{Variety}} + \left(\sum_{e \in \mathcal{E}^{\text{prod}}} \Lambda_{ed} - \sum_{x \in \mathcal{X}^{\text{prod}}} \Lambda_{xd} \right), \tag{C-6}$$

where the second term captures profit income from entering minus exiting producers.

Proof. Each producer's pricing satisfies $p_a = \mu_a \cdot MC_a$, and MC_a depends on P_a^{input}. Writing $\mathrm{d}\log p_a = \mathrm{d}\log \mu_a - \mathrm{d}\log A_a + \mathrm{d}\log P_a^{\text{input}}$ in matrix form:

$$\mathrm{d}\log \mathbf{p} = \mathrm{d}\log \boldsymbol{\mu} - \mathrm{d}\log \mathbf{A} + \tilde{\Omega}\,\mathrm{d}\log \mathbf{p} + \tilde{\Omega}^F\,\mathrm{d}\log \mathbf{w} + \mathbf{V}.$$

Solving: $\mathrm{d}\log \mathbf{p} = \tilde{\Psi}\left(\mathrm{d}\log \boldsymbol{\mu} - \mathrm{d}\log \mathbf{A} + \mathbf{V}\right) + \tilde{\Psi}\tilde{\Omega}^F\,\mathrm{d}\log \mathbf{w}$.

Taking the consumption-weighted average over the continuing basket yields (C-5). Adding the profit-income term from producer entry/exit gives (C-6). This establishes the network variety effect used in Corollary C.1. □

C.1.7 Proof of Corollary B.5

Proof. Equation (B-91) follows from the sector-level CES identity

$$\lambda_{dd,s,t} = \left(\frac{P_{dd,s,t}}{P_{d,s,t}} \right)^{1-\rho},$$

which implies

$$\Delta \log \lambda_{dd,s} = (1-\rho)\big(\Delta \log P_{dd,s} - \Delta \log P_{d,s}\big).$$

Dividing by $\varepsilon = 1-\rho$ and aggregating over sectors with Sato–Vartia weights yields (B-91). Equation (B-92) is immediate from the definition of $\mathcal{A}_d^P$ and the fact that $\sum_s \omega_{ds}^{SV} = 1$. For the channel split, substitute (B-93) into (B-91) and (B-92). This gives

$$\mathcal{A}_d^P = \Delta \log w_d - \sum_{k \in \mathcal{K}_{\text{price}}} \sum_s \omega_{ds}^{SV} p_{k,ds}^{dd},$$

and

$$\mathcal{A}_d^\lambda = \sum_{k \in \mathcal{K}_{\text{price}}} \sum_s \omega_{ds}^{SV} \big(p_{k,ds}^{dd} - p_{k,ds}^{agg}\big),$$

which is exactly (B-94). Adding the two expressions channel by channel yields

$$\mathcal{A}_{k,d}^P + \mathcal{A}_{k,d}^\lambda := -\sum_s \omega_{ds}^{SV} p_{k,ds}^{agg},$$

so each price channel's combined ACR contribution is the negative of its aggregate sector-price contribution. □

C.1.8 Operational First-Order Welfare Decomposition

The following corollary specializes the seven-channel decomposition (Proposition B.13) to the setting relevant for quantitative exercises: no productivity shocks, fixed factor supplies, no exogenous transfers, and stable resident claim rules for continuing income sources. Fixed ownership shares are one sufficient special case.

Corollary C.1 (Operational First-Order Welfare Decomposition)**.** *Assume the following conditions hold.*

(C0) Price-system regularity. *The cost-based Leontief inverse $\tilde{\Psi} = (I - \tilde{\Omega}^{NN})^{-1}$ exists on the continuing producer set $\mathcal{N}^*$ (Proposition C.3).*

(C1) $\mathrm{d}T_d = 0$ *(no change in exogenous transfers; T_d excludes rebated tariff revenue).*

(C2) $\mathrm{d}\log A_a = 0$ *for all producers a (no productivity shocks).*

(C3) Summation domain. *Intensive-margin sums run over $\mathcal{C}_d^{\mathrm{W}}$, the welfare-relevant continuing set: all continuing varieties affecting d's welfare, excluding entrants and exiters (which enter via E_d^{network}).*

(C4) Tariff wedges. *Tariffs τ_{ad} scale purchaser prices faced by all buyers in d (households and firms), so tariff changes enter price effects as an additive $\mathrm{d}\log\tau_{ad}$ wedge (cf. Proposition C.3).*

(C5) $\mathrm{d}\log L_f = 0$ *for all $f \in \mathcal{F}$ (fixed factor supplies). Then the first-order welfare change of destination d is:*

$$\mathrm{d}\log W_d \approx \underbrace{-\sum_{a\in\mathcal{C}_d^{\mathrm{W}}} \tilde{\lambda}_{ad}^{\mathcal{C}}\,\mathrm{d}\log\tau_{ad}}_{\textit{Tariff wedge}} + \underbrace{-\sum_{a\in\mathcal{C}_d^{\mathrm{W}}} \tilde{\lambda}_{ad}^{\mathcal{C}}\,\mathrm{d}\log\mu_a}_{\textit{Markup wedge}}$$
$$\underbrace{-\sum_{b\in\mathcal{B}_d^{C}} \big(\tilde{\Lambda}_{bd}^{\mathcal{C}} - \Lambda_{bd}\big)\mathrm{d}\log\Lambda_b}_{\textit{Factor income wedge (continuing)}} + \underbrace{E_d^{\text{network}}}_{\textit{Network variety effect}}, \tag{C-7}$$

where $\tilde{\lambda}_{ad}^{\mathcal{C}}$ is the cost-based welfare exposure of d to producer a through the Leontief inverse (Proposition C.3), $\tilde{\Lambda}_{bd}^{\mathcal{C}}$ is the corresponding cost-weighted factor-income exposure, Λ_{bd} is destination d's resident-income share from source b, and Λ_b is the world income share of source b (so under the world-GDP numeraire, $\mathrm{d}\log\Lambda_b$ equals the log change in world income from b), $\mathcal{B}_d^{C}$ collects the continuing income claims defined in Corollary B.4, and E_d^{network} is the network variety effect from Proposition C.4 (equation (C-6)). For non-factor income sources $b \in \mathcal{B}_d^{C} \setminus \mathcal{F}$, define $\tilde{\Lambda}_{bd}^{\mathcal{C}} \equiv 0$ so that only primary factors contribute to the continuing price index.

Corollary C.2 (One-factor income correction)**.** *Under Proposition B.7, suppose $N_d = w_d$ is the destination wage, labor endowment is fixed, and nominal income satisfies*

$$E_{d,t} = w_{d,t} L_d + \Pi_{d,t} + \mathrm{TR}_{d,t},$$

where $\Pi_{d,t}$ is aggregate profit income and $\mathrm{TR}_{d,t}$ is tariff revenue. Then the income correction in the extended ACR formula reduces to:

$$\mathcal{A}_d^E = \Delta \log\left(\frac{1 - \tau_{\mathrm{TR},d,0} - \tau_{\Pi,d,0}}{1 - \tau_{\mathrm{TR},d,1} - \tau_{\Pi,d,1}}\right), \quad \tau_{\mathrm{TR},d,t} \equiv \frac{\mathrm{TR}_{d,t}}{E_{d,t}}, \quad \tau_{\Pi,d,t} \equiv \frac{\Pi_{d,t}}{E_{d,t}}. \tag{C-8}$$

This is the ACR benchmark closure: when the tariff-revenue share τ_{TR} and profit share τ_{Π} are constant across equilibria, $\mathcal{A}_d^E = 0$ and the classical ACR formula applies without income correction.

C.1.9 Proof of Corollary C.1

Proof. The proof combines the income and price decompositions already established in this appendix, imposing the simplifying assumptions (C0)–(C5).

Step 1: Welfare decomposition. Write $\mathrm{d}\log W_d = \mathrm{d}\log GNE_d - \mathrm{d}\log P_{W_d}$, and decompose each term into a continuing-variety component and a variety component:

$$\mathrm{d}\log GNE_d = \mathrm{d}\log GNE_d^{\mathrm{Cont.}} + \mathrm{d}\log GNE_d^{\mathrm{Var.}}, \qquad \mathrm{d}\log P_{W_d} = \mathrm{d}\log P_{W_d}^{\mathrm{Cont.}} + \mathrm{d}\log P_{W_d}^{\mathrm{Var.}}.$$

Step 2: Continuing income. Nominal GNE comprises factor income, profit income, tariff revenue, and transfers. Under stable resident claim rules for continuing sources and $\mathrm{d}T_d = 0$ (assumption C1), the continuing part admits the exact source decomposition:

$$\mathrm{d}\log GNE_d^{\mathrm{Cont.}} = \sum_{b \in \mathcal{B}_d^C} \Lambda_{bd}\, \mathrm{d}\log \Lambda_b.$$

Step 3: Continuing price index. Imposing $\mathrm{d}\log A_a = 0$ (C2) and $\mathrm{d}\log L_f = 0$ (C5), and incorporating tariff wedges (C4), the continuing price change follows from Proposition C.3:

$$\mathrm{d}\log P_{W_d}^{\mathrm{Cont.}} = \sum_{a \in \mathcal{C}_d^{\mathrm{W}}} \tilde{\lambda}_{ad}^{\mathcal{C}} \big(\mathrm{d}\log \mu_a + \mathrm{d}\log \tau_{ad}\big) + \sum_{b \in \mathcal{B}_d^C} \tilde{\Lambda}_{bd}^{\mathcal{C}}\, \mathrm{d}\log \Lambda_b.$$

For primary factors $f \in \mathcal{F}$, under fixed endowments (C5) and the world-GDP numeraire we have $\mathrm{d}\log \Lambda_f = \mathrm{d}\log w_f$; for non-factor continuing claims $b \in \mathcal{B}_d^C \setminus \mathcal{F}$, we set $\tilde{\Lambda}_{bd}^{\mathcal{C}} \equiv 0$ by convention so that only primary factors enter the continuing price index.

Step 4: Combine continuing terms. Subtracting price from income:

$$\begin{aligned}
\mathrm{d}\log W_d^{\text{Cont.}} &= \mathrm{d}\log GNE_d^{\text{Cont.}} - \mathrm{d}\log P_{W_d}^{\text{Cont.}} \\
&= -\sum_{a\in\mathcal{C}_d^{\mathrm{W}}} \tilde{\lambda}_{ad}^{\mathcal{C}}\,\mathrm{d}\log\tau_{ad} - \sum_{a\in\mathcal{C}_d^{\mathrm{W}}} \tilde{\lambda}_{ad}^{\mathcal{C}}\,\mathrm{d}\log\mu_a - \sum_{b\in\mathcal{B}_d^{C}} \left(\tilde{\Lambda}_{bd}^{\mathcal{C}} - \Lambda_{bd}\right)\mathrm{d}\log\Lambda_b.
\end{aligned}$$

This yields the first three terms in (C-7).

Step 5: Variety effects. The variety income contribution is, to first order,

$$\mathrm{d}\log GNE_d^{\text{Var.}} \approx \sum_{e\in\mathcal{E}^{\text{prod}}} \Lambda_{ed} - \sum_{x\in\mathcal{X}^{\text{prod}}} \Lambda_{xd},$$

where entrant and exiter profit shares are $O(\varepsilon)$. The variety price component uses the network-propagated Feenstra corrections from Proposition C.4:

$$\mathrm{d}\log P_{W_d}^{\text{Var.}} = V_d + \sum_{a\in\mathcal{N}^*} \tilde{\lambda}_{ad}^{\mathcal{C}}\,V_a.$$

Combining income and price variety terms gives

$$E_d^{\text{network}} = \mathrm{d}\log GNE_d^{\text{Var.}} - \mathrm{d}\log P_{W_d}^{\text{Var.}},$$

which is the fourth term in (C-7).

Adding the continuing-variety welfare (Step 4) and the network variety effect (Step 5) yields (C-7). □

C.1.10 Proof of Corollary C.2

Proof. Set $N_d = w_d$. From Proposition B.7, the income correction is $\mathcal{A}_d^E = \Delta\log(E_d/w_d)$. With $E_{d,t} = w_{d,t}L_d + \Pi_{d,t} + \mathrm{TR}_{d,t}$, we have $w_{d,t}L_d = E_{d,t}\big(1 - \tau_{\Pi,d,t} - \tau_{\mathrm{TR},d,t}\big)$, so

$$\frac{E_{d,t}}{w_{d,t}} = \frac{L_d}{1 - \tau_{\Pi,d,t} - \tau_{\mathrm{TR},d,t}}.$$

Taking log changes and using $\Delta\log L_d = 0$ gives

$$\mathcal{A}_d^E = \Delta\log\left(\frac{1 - \tau_{\mathrm{TR},d,0} - \tau_{\Pi,d,0}}{1 - \tau_{\mathrm{TR},d,1} - \tau_{\Pi,d,1}}\right).$$

When τ_Π and τ_{TR} are constant across equilibria, $\mathcal{A}_d^E = 0$, recovering the classical ACR result. □

D Decomposition Accuracy with Entry and Exit

The exact Sato-Vartia welfare decomposition in Appendices B.2 and B.3 is, by construction, exact for any nested CES economy with arbitrary entry and exit. Standard first-order decompositions, however, can generate large errors when the active set of firmschanges. This section documents those errors in the calibrated six-country model and shows that the exact decomposition works well while the first-order approximations can have larger errors.

We compare three methods. The first is the original Baqaee and Farhi (2024) decomposition, which corresponds to the operational first-order predictor in Corollary C.1, equation (C-7), without accounting for the network variety term E_d^{network}. The second augments original Baqaee and Farhi (2024) decomposition with the variety term, yielding the full first-order approximation in equation (C-7). The third is the exact Sato-Vartia decomposition derived in Appendices B.2 and B.3, which uses Sato-Vartia log-mean weights and exact Feenstra variety corrections at every nest of the CES price tree. For each method, we compute the welfare change implied by the decomposition and report the gap relative to the true simulated welfare change.

Accuracy across model configurations. Figure D-1 reports the relative approximation error for the United States across four model configurations that switch entry and markups on or off. When entry is fixed (FE, FM and FE, VM), all three methods perform well: the active set does not change, so the first-order predictor is close to exact and the variety correction is immaterial. Once firms re-optimize entry and exit (VE, FM and VE, VM), the two first-order methods break down. The Baqaee and Farhi (2024) decomposition without the variety term overstates the welfare loss by more than 100%. Adding the variety term corrects the sign but introduces an error of comparable magnitude in the opposite direction, because the first-order treatment of entry and exit is insufficient for large discrete changes in the active set. The exact Sato-Vartia decomposition, by contrast, remains precise in all four configurations.

Accuracy across countries. Figure D-2 extends the comparison to all six countries under the most demanding configuration: variable entry and variable markups (VE, VM). The pattern is consistent. The Baqaee and Farhi (2024) decomposition produces large positive errors for every country, ranging from roughly 60% for the rest of the world to nearly 200% for China. The first-order approximation with the variety term overshoots in the opposite direction, with errors between −25% and −75% for most countries. The exact Sato-Vartia decomposition remains close to zero throughout. These results confirm that the accuracy advantage of the exact decomposition is not specific to the United States but holds across all countries in the model.

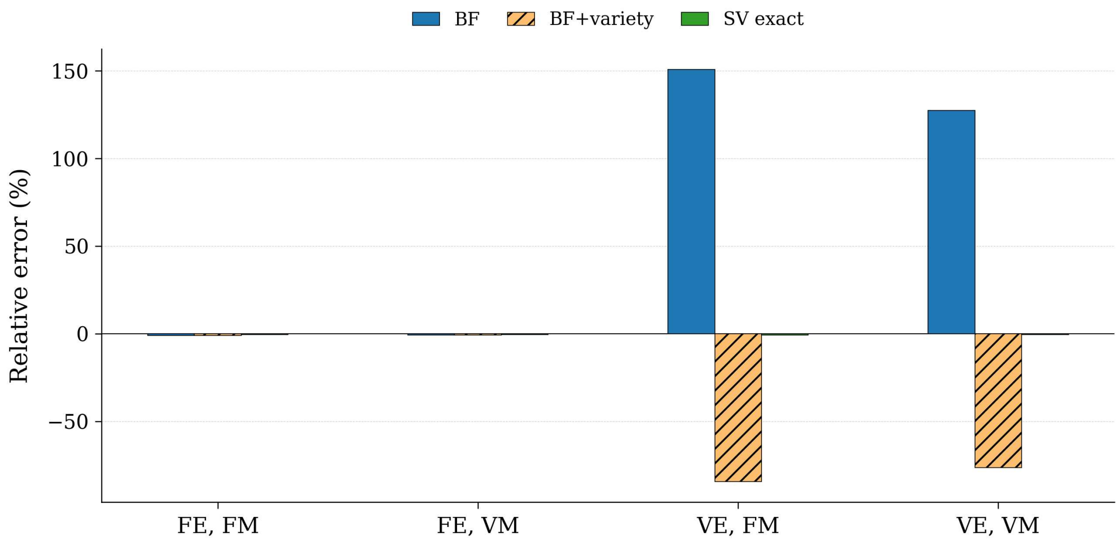

Figure D-1: Decomposition error for the United States across model configurations

Notes: Each bar reports the relative approximation error (percentage of the true welfare change for the United States in the calibrated CHP model. The four configurations vary entry (fixed FE vs. variable VE) and markups (fixed FM vs. variable VM). BF is the Baqaee and Farhi (2024) decomposition (equation (C-7) without E_d^{network}). BF+variety adds the network variety term. SV exact is the Sato-Vartia decomposition from Appendices B.2 and B.3

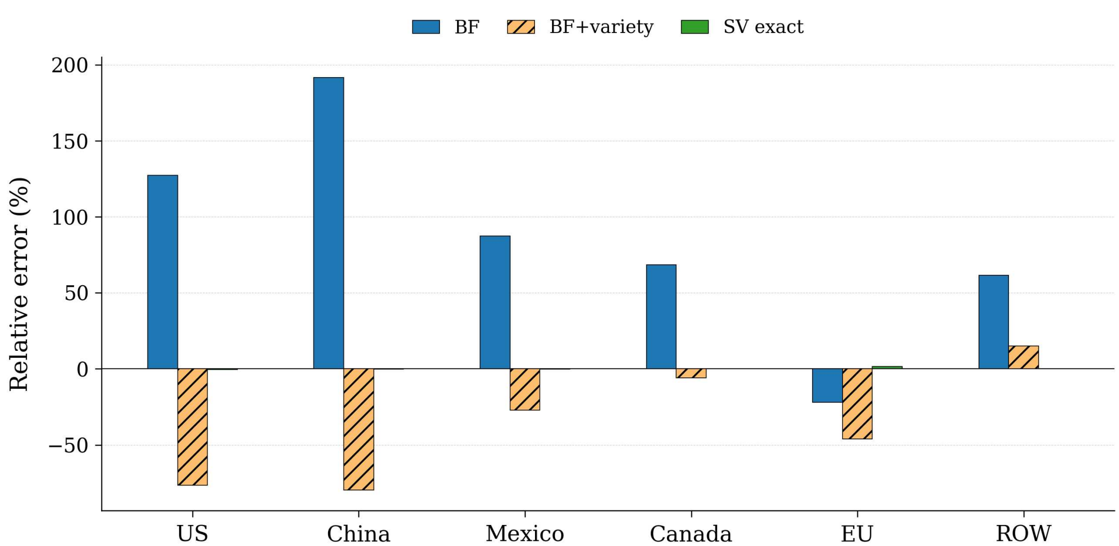

Figure D-2: Decomposition error across countries under variable entry and variable markups

Notes: Each bar reports the relative approximation error (percentage of the true welfare change) under variable entry and variable markups (VE, VM) for the calibrated CHP six-country model. The three methods are as in Figure D-1.

E Quantitative Implementation Details

This section records the computational and calibration details for the quantitative model used in Sections 5 and 6 of the main text. It is an implementation companion, not a new theory section: the exact ex post welfare identities are in Appendices B.2 and B.3; here the focus is equilibrium computation, calibration, and numerical stability.

E.1 Computational Solutions

This subsection summarizes the quantitative solver. We first specify the productivity draws, then record the aggregate equilibrium conditions that close the model, and finally describe the damped fixed-point algorithm used in the computations.

E.1.1 Productivity Dispersion

Firms in all countries draw productivity from a Pareto distribution. The cumulative distribution function of a Pareto random variable with parameters ξ and x_m is

$$F_X(x) = \begin{cases} 1 - (\frac{x_m}{x})^{\xi} & \text{if } x \geq x_m, \\ 0 & \text{otherwise.} \end{cases}$$

We assume $x_m = 1$ (consistent with Edmond, Midrigan and Xu (2015)). The shape parameter ξ governs ex-post productivity dispersion, so lower values of ξ imply greater dispersion.

E.1.2 General Equilibrium Conditions

The equilibrium is a collection of firm-level prices $\{p_{fiodt}\}$, participation decisions $\{\phi_{fiodt}\}$, nominal wages $\{W_{ot}\}$, and aggregate outputs $\{Y_{dt}\}$ such that firms optimize and aggregate resource constraints hold.

Labor market clearing condition:

$$\bar{L}_d = \sum_i \sum_f L_{fidt} = L_{dt}. \tag{E-1}$$

Here $\bar{L}_d$ is country d's inelastic labor endowment, L_{fidt} is firm-level labor demand implied by the Cobb–Douglas production technology, and L_{dt} is aggregate labor demand.

Balanced trade condition:

$$
\begin{aligned}
IMP_{dt} &\equiv \sum_{o\neq d}\sum_{i}\sum_{f} p^{b}_{fiodt} y_{fiodt}, \\
EXP_{dt} &\equiv \sum_{d'\neq d}\sum_{i}\sum_{f} p^{b}_{fidd't} y_{fidd't}, \\
IMP_{dt} &= EXP_{dt}. \qquad \text{(E-2)}
\end{aligned}
$$

The border price is $p^{b}_{fiodt} = p_{fiodt}/\tau_{iodt}$, where τ_{iodt} is one plus the ad valorem tariff. The left-hand side of (E-2) is the value of imports of country d and the right-hand side is the value of exports of country d. In equilibrium, the balanced-trade condition pins down nominal wages up to the choice of numeraire.

E.1.3 Numerical Algorithm

We solve the model with a damped fixed-point iteration on firm-level prices and participation together with aggregate outputs and wages. The algorithm proceeds as follows:

1. Initialize aggregate outputs $\{Y^{(0)}_{dt}\}$, nominal wages $\{W^{(0)}_{ot}\}$, and firm- and origin-level market shares $\{ms^{(0)}_{fiodt}\}$ and $\{ms^{(0)}_{iodt}\}$.

2. Conditional on the current aggregates, update firm-level prices $\{p^{(m+1)}_{fiodt}\}$, participation decisions $\{\phi^{(m+1)}_{fiodt}\}$, and market shares $\{ms^{(m+1)}_{fiodt}\}$ and $\{ms^{(m+1)}_{iodt}\}$ from equations (38)–(41).

3. Aggregate these firm-level objects to obtain updated outputs $\{Y^{(m+1)}_{dt}\}$ from equations (32)–(34), labor demand $\{L^{(m+1)}_{dt}\}$ from (E-1), and imports and exports $\{IMP^{(m+1)}_{dt}, EXP^{(m+1)}_{dt}\}$ from (E-2).

4. Update outputs and nominal wages with damping and continue iterating until labor-market and balanced-trade residuals fall below the convergence tolerances.

5. Stop when the sup norm of changes in firm shares, outputs, and wages falls below 10^{-8}; otherwise continue to the next iteration.

E.2 Calibration

The quantitative model is calibrated to match the production network in the 2014 World Input-Output Database (WIOD), aggregated into six country groups and nine industries. Table E-1 reports the mapping from the 56 WIOD industries to the model's nine categories.

Table E-1: Industry Aggregation: WIOD Industries to Model Categories

Category	Model Industry	WIOD Codes
1	Agriculture & Natural Resources	A01, A02, A03, B
2	Food, Textiles & Basic Manufacturing	C10–C12, C13–C15, C16, C17, C18
3	Heavy & Chemical Manufacturing	C19, C20, C21, C22, C23, C24, C25
4	Electronics & Machinery	C26, C27, C28
5	Transport Equipment Manufacturing	C29, C30
6	Other Manufacturing & Repair	C31_C32, C33
7	Utilities & Construction	D35, E36, E37–E39, F
8	Trade & Transport	G45, G46, G47, H49, H50, H51, H52, H53
9	Knowledge, Public & Personal Services	I, J58, J59_J60, J61, J62_J63, K64, K65, K66, L68, M69_M70, M71, M72, M73, M74_M75, N, O84, P85, Q, R_S, T, U

Notes: Industry codes follow the WIOD 2014 classification. Categories 1–6 are manufacturing and primary sectors; categories 7–9 are services and non-tradable sectors.

The calibration proceeds in two stages. First, we calibrate demand shifters so that the model replicates baseline expenditure shares in final-demand and intermediate-input use. Second, we use the simulated method of moments (SMM) to choose structural parameters that match firm-level tariff elasticities estimated in the data.

E.2.1 Trade Share and Production Network Calibration

We calibrate the demand shifters $(\alpha_{id}, \alpha_{od}, \alpha_{id}^M, \alpha_{od}^M)$ to match the expenditure shares observed in the WIOD 2014 data, where α_{id} and α_{od} are the demand shifters for the industry's share in destination and the origin's share in destination, respectively. The terms α_{id}^M and α_{od}^M are the corresponding demand shifters for intermediate-input demand. The calibration follows an inner-loop-outer-loop procedure.

Inner loop. Given a set of demand shifters, we solve the general equilibrium model by iterating on firms' pricing and entry decisions until all market-clearing conditions are satisfied and firms' market shares and entry decisions converge (see Appendix E.1 for details). The equilibrium yields the trade shares that are then used in the outer loop.

Outer loop. We compare the model-implied trade shares with their empirical counterparts and update demand shifters until the model-implied trade shares are within the convergence tolerances reported below. For each shifter $\alpha \in \{\alpha_{id}, \alpha_{od}\}$, let s^{data} and s^{sim} denote the corresponding empirical and simulated trade shares. The update rule is

$$\alpha^{new} = \phi \cdot \alpha^{old}, \quad \text{where} \quad \phi = \min\left\{5,\ \max\left\{0.2,\ \left(\frac{s^{data}}{s^{sim}}\right)^{\delta}\right\}\right\}. \tag{E-3}$$

The adjustment factor ϕ compares the empirical share with the simulated share. If the empirical share is larger than the simulated share, then the model is under-predicting that share, so the shifter rises. The factor is clamped to $[0.2, 5.0]$ for numerical stability, and $\delta \in (0, 1)$ is a dampening parameter that prevents overshooting. After each update, the relevant block of shifters is renormalized to sum to one within destination, both for final-demand shifters $(\alpha_{id}, \alpha_{od})$ and for intermediate-input shifters $(\alpha^M_{id}, \alpha^M_{od})$.

Because the model includes both final-demand and intermediate-input markets, the outer loop alternates between updating final-demand shifters $(\alpha_{id}, \alpha_{od})$ on even iterations and intermediate-input shifters $(\alpha^M_{id}, \alpha^M_{od})$ on odd iterations. This alternation prevents feedback between the two sets of shifters from destabilizing the algorithm. To further improve stability, we use different dampening factors for final-demand shifters ($\delta = 0.2$) and intermediate-input shifters ($\delta = 0.5$), so that the intermediate-input shares converge more quickly. Fixed export costs are recalibrated periodically whenever the share of active exporters deviates from the target by more than five percentage points, keeping the extensive margin in line with the participation target throughout the calibration.

Convergence criteria. The algorithm converges when the maximum absolute deviation between simulated and empirical shares satisfies the following tolerances simultaneously:

- Industry-destination level (id): $\max_{i,d} |s^{sim}_{id} - s^{data}_{id}| < 0.01$;
- Origin-destination off-diagonal (od, $o \neq d$): $\max_{o \neq d} |s^{sim}_{od} - s^{data}_{od}| < 0.01$;
- Origin-destination diagonal (od, $o = d$): $\max_{o} |s^{sim}_{oo} - s^{data}_{oo}| < 0.03$.

The tighter tolerance on off-diagonal trade shares reflects the fact that foreign shares are smaller and therefore require tighter accuracy.

E.2.2 Parameter Estimation

The four structural parameters reported in Table 2 are chosen by simulated method of moments to match the five tariff elasticities in Table 2. The SMM objective minimizes the weighted absolute deviation

$$\mathcal{L}(\theta) = \sum_{k=1}^{5} w_k \left| m_k^{sim}(\theta) - m_k^{data} \right|, \tag{E-4}$$

where $\theta = (\rho, \sigma, \xi, \varsigma)$ is the parameter vector, $m_k^{sim}(\theta)$ is the k-th model-implied moment, and m_k^{data} is its empirical counterpart. We minimize (E-4) using differential evolution, a population-based global optimizer that is robust to the noisy and non-smooth objective surface generated by repeated full-general-equilibrium simulations.

F Supplementary Quantitative Results

F.1 Comparing Alternative Trade War Counterfactuals

We compare welfare outcomes across three counterfactual tariff scenarios under the calibrated model with production network. Table F-1 summarizes the design. In all three scenarios the US raises bilateral tariffs; scenarios differ in whether and how trading partners respond. In the baseline (scenario 1), all partners retaliate. In scenario 2, only China retaliates while the remaining partners hold tariffs unchanged. Scenario 3 adds a further margin: ROW unilaterally liberalizes, cutting its import tariffs to 3% for all trading partners. Tables F-2 and F-3 report the results. All columns show the VE_VM configuration (variable entry, variable markups); the interaction column reports the difference-in-differences across the four entry–markup configurations within each countergactual scenaios.

Table F-1: Counterfactual Scenario Design

	1: Retaliation	**2: Limited Retal.**	**3: Limited Retal. + ROW Lib.**
USA	30% on CHN, 10% on others	30% on CHN, 10% on others	30% on CHN, 10% on others
CHN	Retaliates	Retaliates	Retaliates
MEX	Retaliates	No retaliation	No retaliation
CAN	Retaliates	No retaliation	No retaliation
EU	Retaliates	No retaliation	No retaliation
ROW	Retaliates	No retaliation	Liberalizes

Notes: US bilateral tariffs are 30% on CHN and 10% on all other partners in every scenario. "Retaliates" means the partner mirrors the US tariff increase. "No retaliation" means the partner keeps baseline tariffs. "Liberalizes" means ROW cuts import tariffs to 3% for all trading partners.

Removing retaliation sharply reduces the welfare cost to the US (Table F-2). Most other countries, however, are worse off: their welfare losses deepen or their welfare gains shrink. The pattern reflects reductions in both the intensive margin and the extensive margin (variety). Countries for which the intensive-margin deterioration from removing retaliation is larger experience the largest welfare declines. Without retaliatory tariffs, trading partners lose tariff revenue and their exporters face US barriers without offsetting protection of their home markets, compressing both margins of adjustment.

When ROW unilaterally liberalizes (scenario 3 versus 2), ROW itself moves from a near-zero welfare change to a welfare loss (Table F-2). The underlying channels partially offset: the intensive-margin loss deepens while the variety gain rises. The intensive-margin deterioration is primarily a terms-of-trade effect—ROW's nominal wage falls sharply (Table F-3)—as lower tariffs reduce the demand for domestic factors. Other countries gain from ROW's liberalization, as lower ROW barriers expand export opportunities for their firms.

Table F-2: Welfare Decomposition across Trade War Scenarios — Baseline Calibration, PN (%)

	Retaliation		**Limited Retal.**		**Limited Retal.+ ROW Lib.**	
	VE_VM	**Int**	VE_VM	**Int**	VE_VM	**Int**
USA						
$\Delta \log Y$	-1.26	**0.25**	-0.31	**0.25**	-0.21	**0.23**
Intensive	2.19	**-1.27**	1.95	**-0.89**	2.03	**-0.79**
Variety	-3.46	**1.53**	-2.34	**1.07**	-2.25	**1.06**
CHN						
$\Delta \log Y$	-0.12	**0.08**	-0.17	**0.06**	-0.14	**0.07**
Intensive	0.28	**-0.05**	0.32	**-0.06**	0.34	**-0.03**
Variety	-0.41	**0.14**	-0.48	**0.16**	-0.47	**0.13**
MEX						
$\Delta \log Y$	-0.08	**0.67**	-0.34	**0.48**	-0.01	**0.65**
Intensive	0.26	**-2.77**	-0.32	**-2.52**	-0.18	**-2.39**
Variety	-0.38	**3.28**	-0.06	**2.91**	0.28	**3.10**
CAN						
$\Delta \log Y$	-0.39	**2.53**	-0.31	**2.71**	-0.25	**2.72**
Intensive	-0.01	**-1.76**	-0.37	**-1.50**	-0.42	**-1.59**
Variety	-0.38	**4.63**	0.15	**4.37**	0.22	**4.36**
EU						
$\Delta \log Y$	0.05	**0.14**	0.01	**0.14**	0.09	**0.15**
Intensive	0.02	**-0.06**	-0.08	**-0.08**	-0.03	**-0.06**
Variety	0.03	**0.20**	0.09	**0.25**	0.10	**0.17**
ROW						
$\Delta \log Y$	0.11	**0.16**	0.00	**0.16**	-0.08	**0.15**
Intensive	0.00	**-0.05**	-0.08	**-0.03**	-0.45	**0.00**
Variety	0.10	**0.20**	0.09	**0.21**	0.36	**0.16**

Notes: Baseline calibration with production network ($\ell = 0.6$). Values in percentage points. $\Delta \log Y$ = Intensive + Variety. **Int** = Interaction: VE_VM − FE_VM − VE_FM + FE_FM. Results are medians over 5 seeds.

Table F-3: Aggregate Statistics across Trade War Scenarios — Baseline Calibration, PN (%)

	Retaliation		Limited Retal.		Limited Retal.+ ROW Lib.	
	VE_VM	**Int**	VE_VM	**Int**	VE_VM	**Int**
USA						
$\Delta \log W$	-3.27	**-0.51**	1.58	**-0.78**	2.36	**-0.82**
GNI	-1.41	**-0.32**	3.22	**-0.68**	3.91	**-0.74**
Wage	-2.02	**-0.32**	0.95	**-0.47**	1.41	**-0.49**
Profit	-0.08	**-0.07**	1.44	**-0.27**	1.64	**-0.31**
Tariff rev.	0.68	**0.07**	0.84	**0.06**	0.86	**0.06**
CHN						
$\Delta \log W$	-0.02	**0.45**	-0.74	**0.55**	-0.63	**0.50**
GNI	0.10	**0.52**	-0.63	**0.56**	-0.53	**0.58**
Wage	-0.01	**0.27**	-0.45	**0.34**	-0.38	**0.30**
Profit	0.10	**0.19**	-0.16	**0.23**	-0.12	**0.24**
Tariff rev.	0.00	**0.01**	-0.02	**0.01**	-0.02	**0.01**
MEX						
$\Delta \log W$	-0.78	**-4.31**	-1.07	**-3.96**	-0.41	**-4.04**
GNI	0.00	**-4.35**	-0.98	**-3.96**	-0.32	**-3.97**
Wage	-0.50	**-2.72**	-0.69	**-2.53**	-0.26	**-2.56**
Profit	-0.10	**-1.70**	-0.38	**-1.56**	-0.17	**-1.57**
Tariff rev.	0.53	**0.08**	0.01	**0.02**	0.03	**0.02**
CAN						
$\Delta \log W$	-1.06	**-2.97**	-1.14	**-2.84**	-0.72	**-2.73**
GNI	-0.42	**-3.02**	-1.20	**-3.08**	-0.76	**-2.99**
Wage	-0.68	**-1.88**	-0.73	**-1.80**	-0.46	**-1.72**
Profit	-0.26	**-1.19**	-0.43	**-1.20**	-0.30	**-1.20**
Tariff rev.	0.53	**0.10**	0.01	**0.04**	0.02	**0.04**
EU						
$\Delta \log W$	0.50	**0.25**	-0.20	**0.21**	0.23	**0.18**
GNI	0.54	**0.27**	-0.23	**0.24**	0.17	**0.20**
Wage	0.30	**0.16**	-0.12	**0.13**	0.14	**0.11**
Profit	0.18	**0.09**	-0.11	**0.10**	0.03	**0.09**
Tariff rev.	0.05	**0.01**	0.00	**0.00**	0.00	**0.00**
ROW						
$\Delta \log W$	0.21	**0.54**	-0.30	**0.57**	-0.83	**0.61**
GNI	0.23	**0.54**	-0.35	**0.56**	-1.24	**0.56**
Wage	0.13	**0.33**	-0.18	**0.35**	-0.51	**0.37**
Profit	0.05	**0.20**	-0.17	**0.21**	-0.50	**0.21**
Tariff rev.	0.05	**0.01**	0.00	**0.01**	-0.22	**0.01**

Notes: Values in percentage points. $\Delta \log W$ = nominal wage. GNI components use the Sato–Vartia (log-mean) decomposition: Wage + Profit + Tariff rev. = $\Delta \log$ GNI exactly. **Int** = Interaction: VE_VM − FE_VM − VE_FM + FE_FM. Results are medians over 5 seeds.

Tables F-4 and F-5 decompose the welfare channels underlying the cross-shock comparison. Each entry reports the difference relative to the retaliation baseline (scenario 1), so positive values indicate a larger welfare component under the alternative scenario. The variety-income margin for MEX and CAN changes dramatically across scenarios. When these countries do not retaliate, their exporters face cheaper imported inputs—since retaliatory tariffs on their own imports are no longer in place—which helps them stay active in foreign markets. Reduced exit translates directly into smaller variety-income losses: the wage bills and profits of exiting exporters that would otherwise vanish from household income are partially preserved.

On the intensive margin, removing retaliation mechanically shrinks the tariff wedge for MEX and CAN—they no longer impose tariffs that distort their own consumer prices. But the allocative factor moves against them: without retaliatory tariffs shielding the home market, domestic firms lose market share, wages fall, and profits contract. The wage and profit losses through the allocative factor more than offset the direct price benefit of lower tariff wedges, so the intensive margin deteriorates on net. Combining both margins, retaliation leaves MEX and CAN better off overall—the home-market protection it provides outweighs the input-cost burden it imposes on their exporters.

Table F-4: Retaliation vs. Limited Retaliation: Δ Welfare Decomposition (%, VE_VM)

	Δ USA	Δ CHN	Δ MEX	Δ CAN	Δ EU	Δ ROW
$\Delta \log Y$	0.95	-0.05	-0.26	0.08	-0.04	-0.10
Intensive	-0.23	0.04	-0.58	-0.37	-0.10	-0.08
Alloc wedge	0.20	0.01	0.69	0.77	0.09	0.07
Markup	0.26	-0.01	0.05	0.14	0.01	-0.01
dom	0.20	-0.01	0.03	0.03	0.01	0.00
for	0.07	0.00	0.02	0.09	0.00	0.00
Tariff wedge	-0.07	0.02	0.64	0.66	0.09	0.07
Alloc factor	-0.43	0.04	-1.24	-1.16	-0.20	-0.14
AF buy	-4.29	0.69	-0.06	-0.23	0.61	0.42
AF sell	3.87	-0.68	-1.03	-0.99	-0.78	-0.59
Wage	2.34	-0.41	-0.32	-0.30	-0.43	-0.32
dom	2.02	-0.38	-0.47	-0.48	-0.42	-0.29
exp	0.31	-0.03	0.16	0.12	-0.01	-0.03
Profit	1.41	-0.26	-0.26	-0.19	-0.29	-0.22
dom	1.28	-0.24	-0.32	-0.31	-0.29	-0.19
exp	0.12	-0.01	0.06	0.06	0.00	-0.01
Tariff	0.13	-0.01	-0.50	-0.50	-0.06	-0.05
Variety	1.12	-0.07	0.32	0.54	0.07	-0.01
Price	0.40	-0.03	0.06	0.28	0.03	-0.01
dom	0.00	0.00	0.00	-0.01	0.00	0.00
for	0.40	-0.03	0.06	0.29	0.03	-0.01
Income	0.73	-0.05	0.23	0.26	0.04	0.00
Wage	0.62	-0.04	0.26	0.22	0.04	0.01
dom	0.00	0.00	0.00	-0.03	0.00	0.00
exp	0.62	-0.04	0.26	0.25	0.04	0.01
Profit	0.11	0.00	0.00	0.02	0.00	-0.01
dom	0.00	0.00	0.00	0.00	0.00	0.00
exp	0.11	0.00	0.00	0.02	0.00	-0.01
Tariff	0.02	0.00	-0.03	-0.02	0.00	0.00

Notes: Baseline calibration with production network ($\ell = 0.6$), VE_VM. Values show the difference (Limited Retaliation − Retaliation) in percentage points. Positive values mean higher under the Limited Retaliation scenario. $\Delta \log Y$ = Intensive + Variety. Intensive = Alloc wedge + Alloc factor. Alloc wedge = Markup (dom + for) + Tariff wedge. Alloc factor = AF_buy + AF_sell. AF_sell = Wage (dom + exp) + Profit (dom + exp) + Tariff. Variety = Price (dom + for) + Income. Income = Wage (dom + exp) + Profit (dom + exp) + Tariff. Medians over 5 seeds.

Table F-5: Retaliation vs. Limited Retaliation + ROW Liberalizes: Δ Welfare Decomposition (%, VE_VM)

	Δ USA	Δ CHN	Δ MEX	Δ CAN	Δ EU	Δ ROW
$\Delta \log Y$	1.05	-0.02	0.08	0.14	0.04	-0.19
Intensive	-0.15	0.06	-0.44	-0.41	-0.05	-0.45
Alloc wedge	0.21	0.03	0.75	0.75	0.12	0.39
Markup	0.29	0.00	0.10	0.11	0.03	0.05
dom	0.22	0.00	0.08	0.03	0.03	0.04
for	0.07	0.00	0.04	0.05	0.00	0.01
Tariff wedge	-0.08	0.03	0.65	0.66	0.09	0.34
Alloc factor	-0.40	0.04	-1.20	-1.16	-0.17	-0.84
AF buy	-5.00	0.61	-0.56	-0.61	0.25	0.83
AF sell	4.48	-0.59	-0.58	-0.55	-0.39	-1.67
Wage	2.71	-0.35	-0.03	0.00	-0.19	-0.81
dom	2.31	-0.34	-0.26	-0.26	-0.25	-0.85
exp	0.40	-0.01	0.30	0.20	0.06	0.05
Profit	1.63	-0.23	-0.11	-0.06	-0.15	-0.56
dom	1.46	-0.23	-0.20	-0.16	-0.19	-0.61
exp	0.17	0.00	0.13	0.09	0.03	0.04
Tariff	0.14	-0.01	-0.50	-0.49	-0.06	-0.28
Variety	1.21	-0.06	0.66	0.60	0.08	0.26
Price	0.46	-0.04	0.26	0.26	0.04	0.06
dom	0.00	0.00	0.00	-0.01	0.00	0.00
for	0.46	-0.04	0.27	0.28	0.04	0.06
Income	0.76	-0.04	0.36	0.28	0.05	0.20
Wage	0.63	-0.04	0.31	0.25	0.05	0.19
dom	0.00	0.00	0.00	-0.03	0.00	0.00
exp	0.63	-0.04	0.32	0.27	0.05	0.19
Profit	0.10	0.00	0.03	0.02	0.00	0.01
dom	0.00	0.00	0.00	0.00	0.00	0.00
exp	0.10	0.00	0.03	0.02	0.00	0.01
Tariff	0.03	0.00	-0.01	-0.01	0.00	0.00

Notes: Baseline calibration with production network ($\ell = 0.6$), VE_VM. Values show the difference (Limited Retal. + ROW Lib. − Retaliation) in percentage points. Positive values mean higher under the alternative scenario. $\Delta \log Y$ = Intensive + Variety. Intensive = Alloc wedge + Alloc factor. Alloc wedge = Markup (dom + for) + Tariff wedge. Alloc factor = AF_buy + AF_sell. AF_sell = Wage (dom + exp) + Profit (dom + exp) + Tariff. Variety = Price (dom + for) + Income. Income = Wage (dom + exp) + Profit (dom + exp) + Tariff. Medians over 5 seeds.

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