

Trade Policy Uncertainty and Optimal Monetary Policy

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Trade policy has become a prominent source of macroeconomic risk



Key Questions

- How does trade policy uncertainty transmit through the economy?
- What determines the size and direction of the macroeconomic response?
- How does monetary policy affect transmission, and what policy is optimal?

Existing work on uncertainty shocks typically relies on numerical simulations
⇒ limited analytical characterization of transmission and policy

This Paper

Analytically characterize the baseline transmission of export-tariff uncertainty shocks in SOE:

1. **Closed-form IRFs** with two high-order wedges: **NKPC** vs **UIP**
 - opposing effects on output: can be either expansionary or contractionary
 - same direction on inflation: large deflationary impacts

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Implication: Monetary policy shapes the magnitude *and* direction of macro responses to tariff uncertainty shocks.

Model: SOE NK with Tariff Stochastic Volatility

Small open-economy New Keynesian model (*a la* Galí & Monacelli 2005)

- Rotemberg price adjustment costs, producer currency pricing
- Complete international asset markets (incomplete markets as extension)
- Monetary policy: Taylor rule with PPI inflation ($r_t = \phi_\pi \pi_t + \phi_y y_t$), or Ramsey

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Export tariff τ_t with **stochastic volatility** $\sigma_{T,t}$

$$\tau_t = \rho_\tau \tau_{t-1} + e^{\sigma_{T,t-1}} \varepsilon_{\tau,t}$$

$$\sigma_{T,t} = \rho_\sigma \sigma_{T,t-1} + \varepsilon_{\sigma,t}$$

- Uncertainty shock $\varepsilon_{\sigma,t}$ moves future $\sigma_{T,t}$, while keeping the level fixed ($\varepsilon_{\tau,t} = 0$)

Minimal 2-equation System

- Equilibrium collapses to two forward-looking equations in PPI inflation π_t and the terms of trade q_t (\uparrow means real depreciation)
- Tariff-volatility shocks introduce expectations wedges:

$$\begin{aligned}\pi_t &= \kappa_{mc} mc_t(q_t) + \beta \mathbb{E}_t[\pi_{t+1}] + \Delta_t^{NK} + O(4) \\ r_t(\pi_t, q_t) &= \mathbb{E}_t[(q_{t+1} - q_t) + \pi_{t+1}] + \Delta_t^{UIP} + O(4)\end{aligned}$$

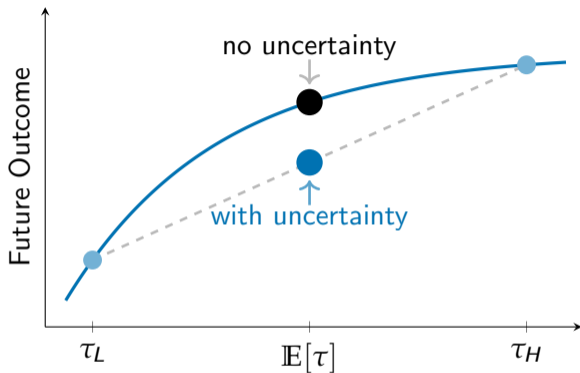
where

$$\Delta_t^{NK} = u^{NK} \sigma_{T,t} + O(4), \quad \Delta_t^{UIP} = u^{UIP} \sigma_{T,t} + O(4)$$

\Rightarrow Baseline signs: ($u^{NK} < 0, u^{UIP} < 0$); two wedge loadings

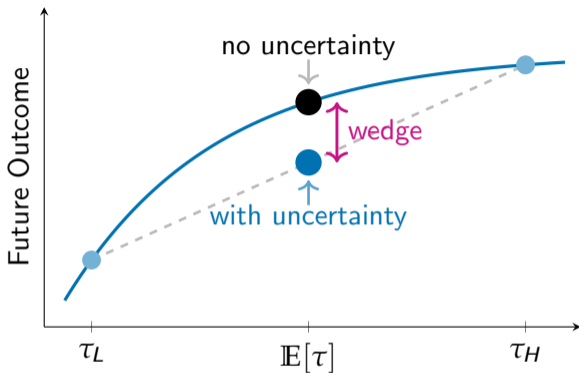
Lemma 1

Intuition: Why Wedges Arise and What They Depend On



- Uncertainty alters expected values of future outcomes (e.g. $\mathbb{E}[\pi_{t+1}]$ and $\mathbb{E}[q_{t+1}]$)
- In general, the sign of the wedge depends on the distribution of future outcomes (e.g. whether it is convex or concave in tariff states)

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Pricing Wedge u^{NK}

How does tariff risk affect pricing incentives?

$$u^{NK} \propto \underbrace{3A_{\pi}^2}_{\text{Rotemberg cost} > 0} + \underbrace{2A_{\pi}s}_{\text{Covariance} < 0} + \underbrace{B_{\pi}}_{\text{Curvature} < 0}$$

where

- $A_{\pi} > 0$ and $B_{\pi} < 0$ govern the response of inflation to a tariff **level** shock:

$$\pi_t = A_{\pi}\tau_t + \frac{1}{2}B_{\pi}\tau_t^2$$

- $A_{\pi}s$ captures how inflation co-variates with discount factor s

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Intuition:

- Risk creates inflation dispersion, raising expected adjustment costs
 - Low tariffs mean high sales, so price deviations matter more there
- ⇒ Firms weight low-tariff states more, pulling prices down; concavity reinforces this

UIP / Risk-Premium Wedge u^{UIP}

How does tariff risk affect exchange rates?

$$u^{UIP} \propto \underbrace{-(A_\pi + A_q)^2}_{\text{Variance} < 0} + \underbrace{(B_\pi + B_q)}_{\text{Curvature} < 0}$$

where (A_π, B_π) and (A_q, B_q) govern the responses of inflation and the terms of trade to tariff **level** shocks:

$$\pi_t = A_\pi \tau_t + \frac{1}{2} B_\pi \tau_t^2 \quad \text{and} \quad q_t = A_q \tau_t + \frac{1}{2} B_q \tau_t^2$$

Intuition:

- ⇒ Tariff risk increases uncertainty about future exchange rates
- ⇒ Foreign-currency assets become riskier in home-currency terms
- ⇒ Appreciation of home currency today to raise expected return

Closed-Form Uncertainty Responses

IRFs to tariff uncertainty shock:

$$\pi_t = \rho_\sigma^t \chi_\pi, \quad q_t = \rho_\sigma^t \chi_q, \quad y_t = \theta_q q_t,$$

where ρ_σ is persistence of uncertainty shock and

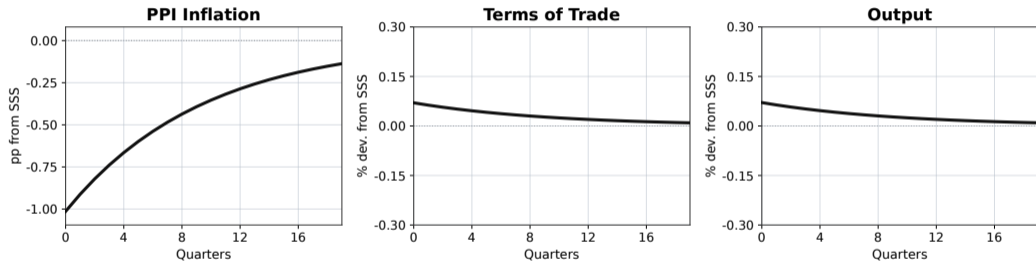
$$\chi_\pi \propto \overbrace{(\phi_y \theta_q + 1 - \rho_\sigma) u^{NK}}^{(-)} + \overbrace{\tilde{\kappa} u^{UIP}}^{(-)} \quad \chi_q \propto \overbrace{-(\phi_\pi - \rho_\sigma) u^{NK}}^{(+)} + \overbrace{(1 - \beta \rho_\sigma) u^{UIP}}^{(-)}$$

- $\theta_q \approx 1$ under baseline calibration
- ϕ_π and ϕ_y are Taylor rule parameters for inflation and output

Responses to Export Tariff Uncertainty

Shock: doubling tariff volatility

— Baseline - - - NK only ($u^{UIP} = 0$) - · - · UIP only ($u^{NK} = 0$)

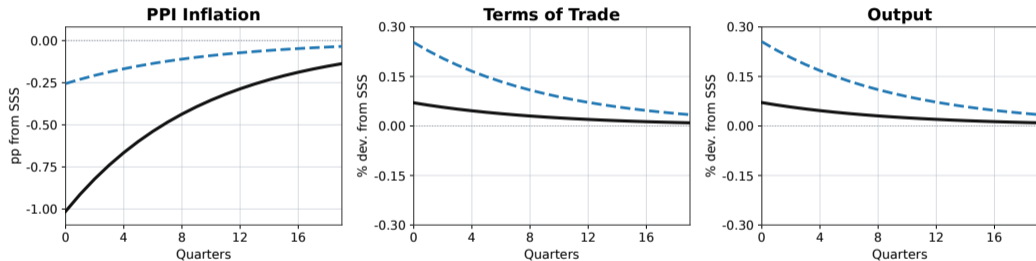


- Large deflationary effects, but small impacts on terms of trade and output

Responses to Export Tariff Uncertainty

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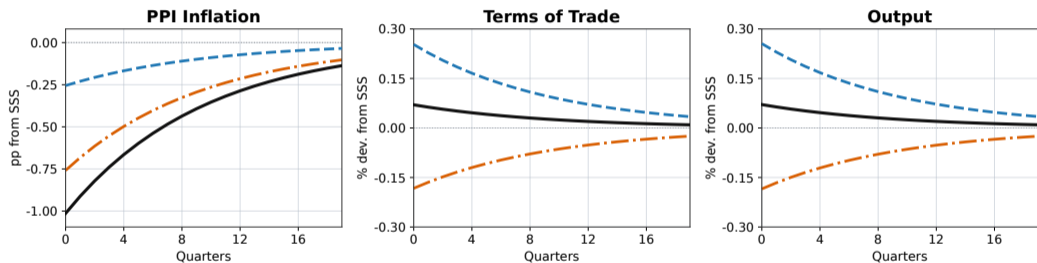


- **NKPC-only:** deflation triggers easing, real depreciation and higher output

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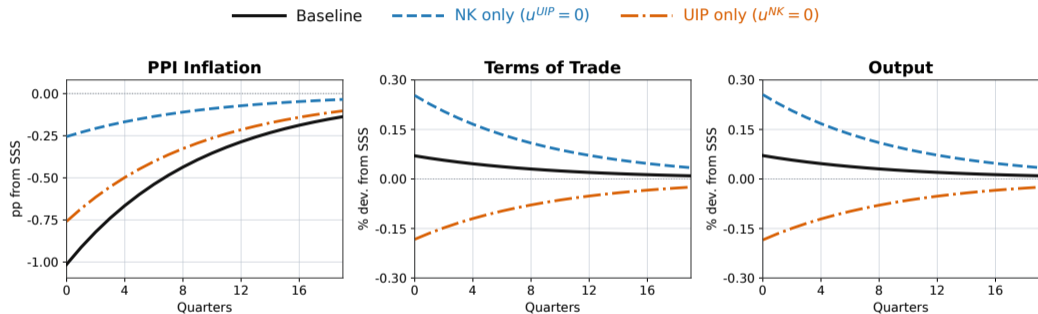
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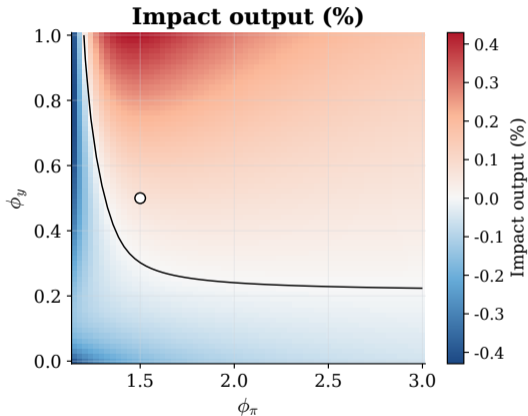
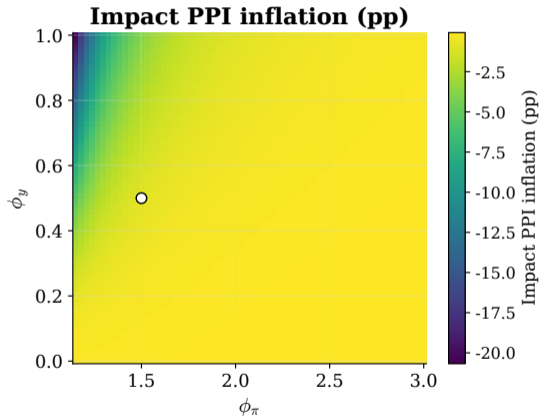
Responses to Export Tariff Uncertainty

Shock: doubling tariff volatility



- **NKPC-only:** deflation triggers easing, real depreciation and higher output
- **UIP-only:** asset-pricing effects drive appreciation and lower output
- **Together:** both channels are deflationary, but they offset on the real side

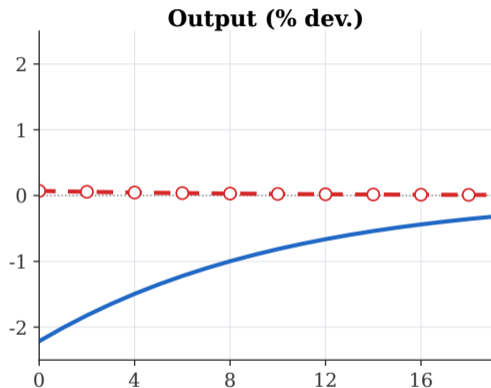
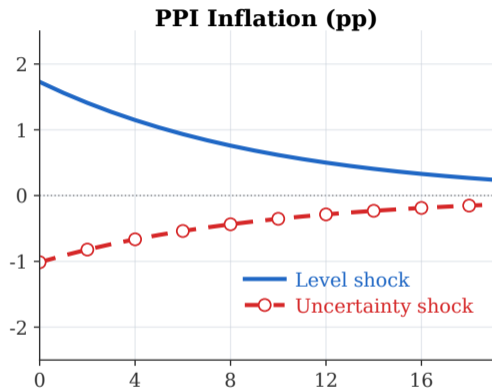
Uncertainty shocks can be expansionary or contractionary depending on monetary policy



- Contractionary when monetary policy looks through uncertainty shocks (low ϕ_π or ϕ_y)

Opposite Effects of Level vs Uncertainty Shocks

Responses to a 10% rise in export tariffs versus a doubling of tariff uncertainty



- Uncertainty can dampen the effects of level shocks

Optimal Ramsey Policy

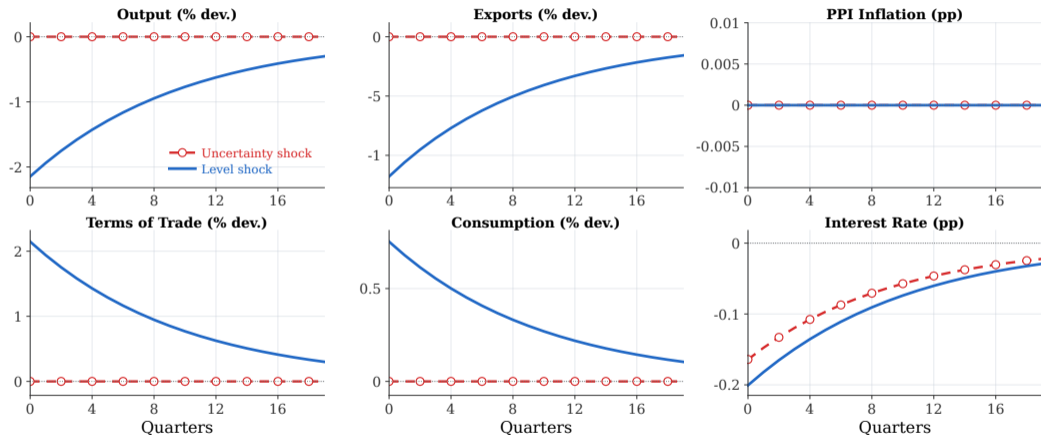
Divine coincidence for tariff uncertainty

Under complete markets with producer currency pricing, **strict PPI inflation targeting** ($\pi_{H,t} = 0 \forall t$) is Ramsey optimal and makes real effects of uncertainty shocks $O(4)$.

- Strict PPI inflation targeting fully stabilizes current and future prices $\pi_{H,t} = 0$, which eliminates price-adjustment frictions and hence output gap $\tilde{y}_t = 0$
- Benchmark flexible-price allocation depends on tariff *levels*, not volatility
- UIP wedge adjusts the interest rate, leaving real allocations unaffected

Same Optimal Policy, Different Macro Responses

Responses to tariff level vs uncertainty shocks under strict PPI targeting



- Optimal policy implies full stabilization of uncertainty shocks, while output falls under level shocks.

Conclusion

Study the baseline transmission of export-tariff uncertainty shocks in SOE:

1. **Two-wedge system:** NKPC + UIP pin down uncertainty IRFs

⇒ small output effects but large deflationary impacts

⇒ uncertainty can dampen the effects of level shocks

2. **Optimal policy:** Complete market ⇒ strict PPI targeting

Implication: Monetary policy plays a crucial role in shaping both magnitude and direction of macro responses to uncertainty shocks

Appendix

A1: Baseline Calibration

Parameter	Description	Value
β	Discount factor	0.99
σ	Risk aversion	2
φ	Inverse Frisch elasticity	1
α	Import share (openness)	0.30
γ	Trade elasticity (Armington)	1.5
ε	Elasticity of substitution	10
κ	Rotemberg adjustment cost	40
ϕ_π	Taylor rule: inflation	1.5
ϕ_y	Taylor rule: output gap	0.5
ρ_τ	Tariff persistence	0.9
$\bar{\sigma}_\tau$	Steady-state tariff volatility	0.10
ρ_σ	Volatility persistence	0.9
η_σ	Volatility shock size	$\log 2 \approx 0.693$

A2: Key Equilibrium Conditions

$$\text{NKPC: } \kappa(\Pi_{H,t} - 1)\Pi_{H,t} = (\varepsilon - 1)(mc_t - 1) + \beta\kappa \mathbb{E}_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} (\Pi_{H,t+1} - 1)\Pi_{H,t+1} \frac{Y_{t+1}}{Y_t} \right]$$

$$\text{UIP: } \mathbb{E}_t \left[\beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{Q_t}{Q_{t+1}} \frac{1 + r_t}{1 + \pi_{C,t+1}} \right] = 1$$

$$\text{Taylor: } 1 + r_t = \beta^{-1}(1 + \pi_{H,t})^{\phi_\pi} \left(\frac{Y_t}{Y_{SS}} \right)^{\phi_y}$$

$$\text{Tariff SV: } \tau_t = \rho_\tau \tau_{t-1} + \bar{\sigma}_\tau e^{\sigma_\tau, t-1} \varepsilon_{\tau, t}, \quad \sigma_{T, t} = \rho_\sigma \sigma_{T, t-1} + \eta_\sigma \varepsilon_{\sigma, t}$$

Lemma 1: Exact Nonlinear 2×2 System

Under CM + PCP + risk sharing, equilibrium reduces to two equations in $(\Pi_{H,t}, Q_t)$:

$$\kappa(\Pi_{H,t} - 1)\Pi_{H,t} = (\epsilon - 1)(mc_t - 1) + \beta\kappa\mathbb{E}_t \left[(\Pi_{H,t+1} - 1)\Pi_{H,t+1} \frac{Y_{t+1}}{Y_t} \frac{Q_t}{Q_{t+1}} \right]$$

$$1 = \Pi_{H,t}^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} e^{\psi_t} \mathbb{E}_t \left[\frac{Q_t}{Q_{t+1} \Pi_{H,t+1}} \right]$$

- Output and marginal cost are **static functions** of $(\Pi_{H,t}, Q_t, \tau_t)$
- Risk sharing: $C_t = (G_t/Q_t)^{-1/\sigma}$ eliminates Euler equation

The $\mathbf{M}(\rho)$ Template and Static Elasticities

All three coefficient layers solve a 2×2 linear system with the **same matrix**:

$$\mathbf{M}(\rho) \equiv \begin{pmatrix} 1 - \beta\rho & -\tilde{\kappa} \\ \phi_\pi - \rho & \phi_y\theta_q + 1 - \rho \end{pmatrix}, \quad \tilde{\kappa} \equiv \kappa_{mc}(1 + \varphi\theta_q)$$

Three layers, one template:

- (A_π, A_q) : first-order tariff $\rightarrow \mathbf{M}(\rho_\tau)$
- (B_π, B_q) : second-order curvature $\rightarrow \mathbf{M}(\rho_\tau^2)$
- (χ_π, χ_q) : uncertainty $\rightarrow \mathbf{M}(\rho_\sigma)$

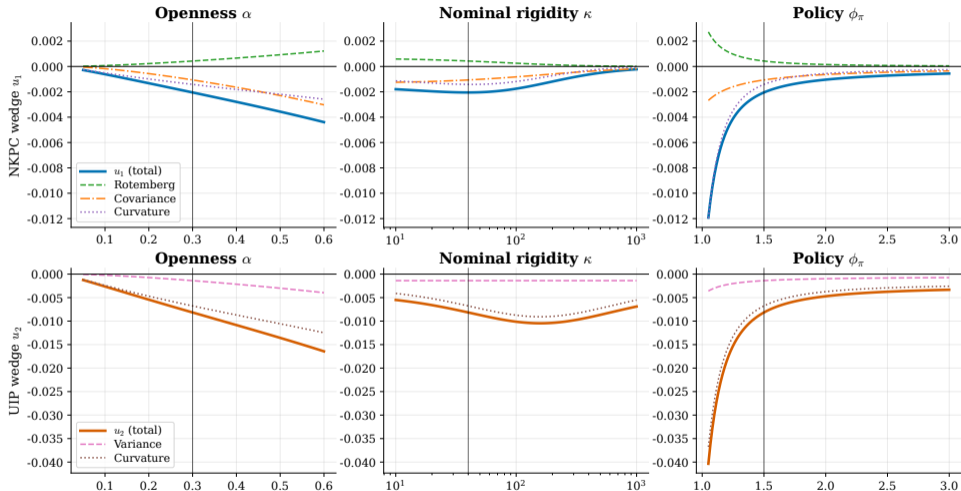
Static elasticities:

$$\theta_q = (1 - \alpha) \left[\frac{1}{\sigma} + \alpha \left(\gamma - \frac{1}{\sigma} \right) \right] + \alpha\gamma$$

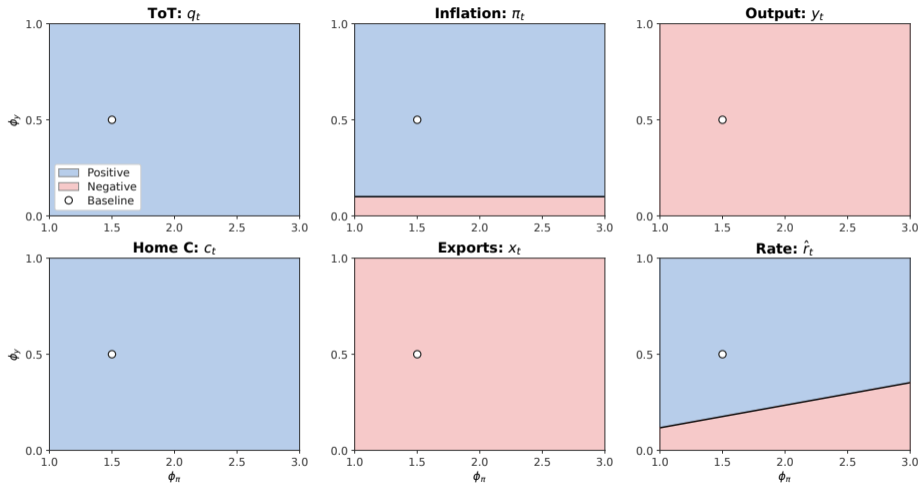
$$\theta_\tau = \alpha\gamma$$

$$\kappa_{mc} = (\epsilon - 1) / \kappa$$

Wedge Component Sensitivity

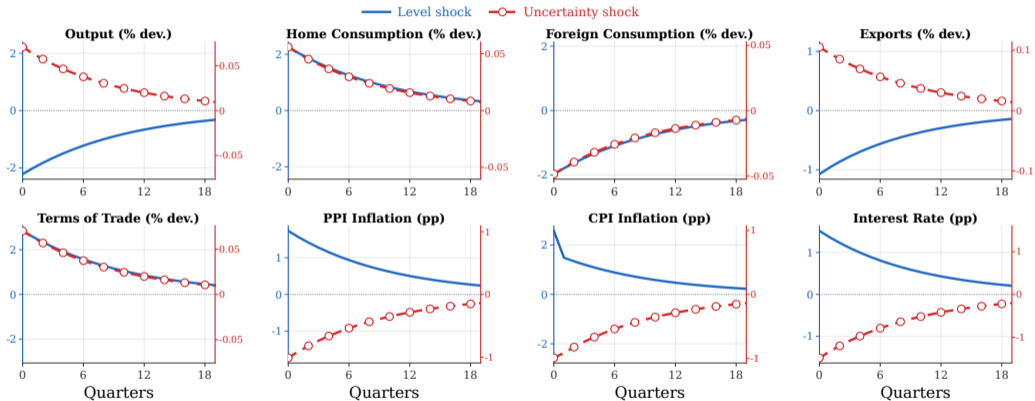


Level-Shock Sign Regions



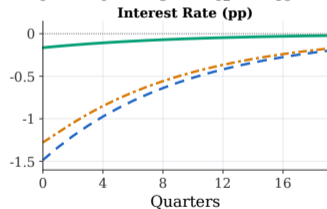
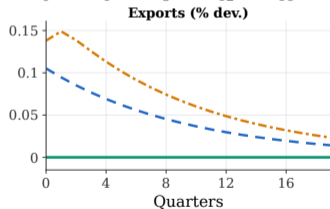
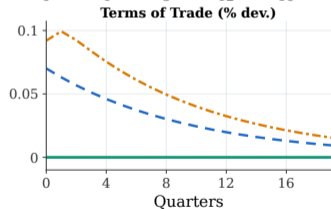
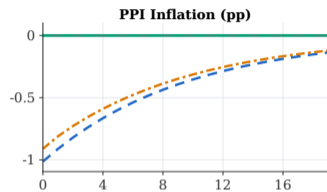
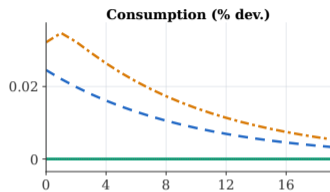
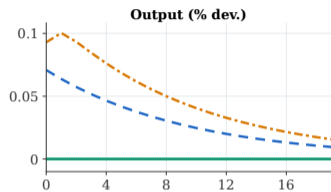
Opposite Responses

All Variables: Level vs Uncertainty



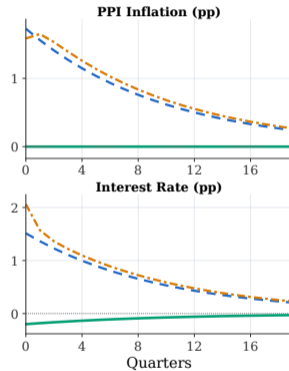
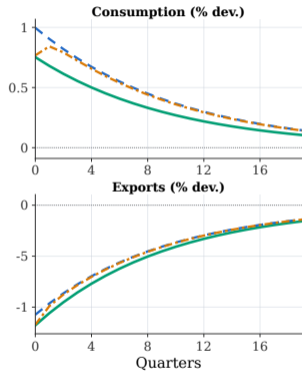
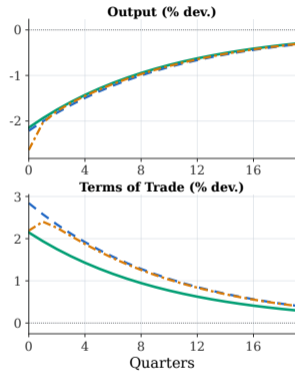
Opposite Responses

Responses to Tariff Uncertainty Shock under Alternative MP Rules



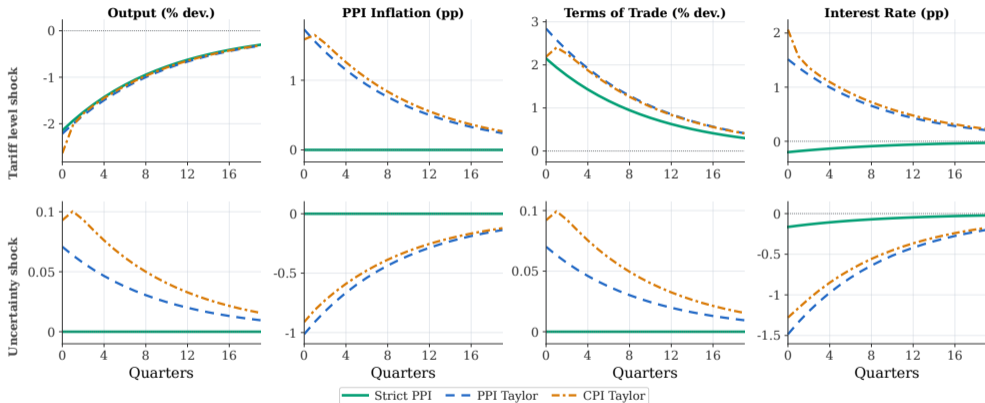
— Strict PPI - - - PPI Taylor - . - CPI Taylor

Policy Comparison: Tariff Level Shock



— Strict PPI - - PPI Taylor - - CPI Taylor

Policy Comparison: Level and Uncertainty



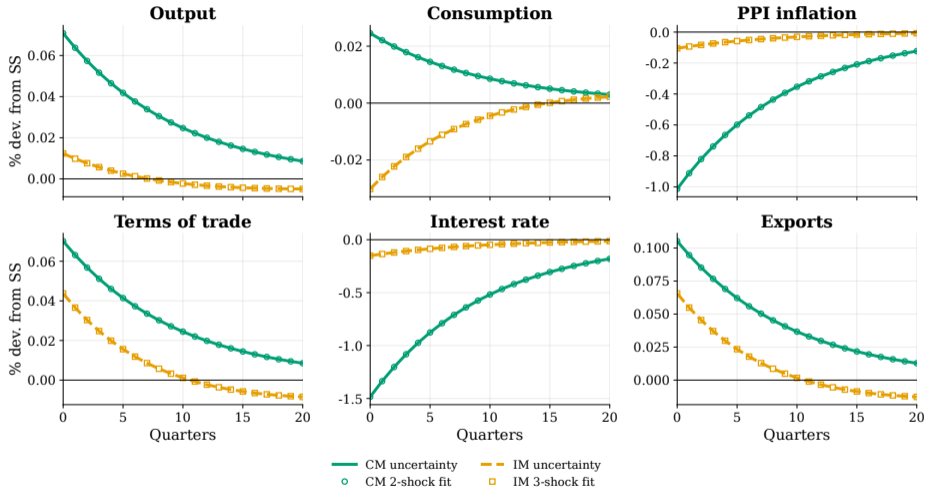
Incomplete Markets: Precautionary-savings Channel

Without risk sharing, the Euler equation gives rise to the *precautionary-savings channel*:

$$\hat{c}_t = \mathbb{E}_t[\hat{c}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - \mathbb{E}_t[\hat{\pi}_{C,t+1}]) - \frac{1}{\sigma} \tilde{\Delta}_t^E + O(4)$$

- Euler/Jensen wedge: $\tilde{\Delta}_t^E \propto \bar{\sigma}_\tau^2 (e^{2\sigma\tau,t} - 1)$
- 3×3 system: NKPC + UIP + Euler
- Divine coincidence **breaks**: PPI targeting $\not\Rightarrow \tilde{y}_t = 0$ generically

Responses to Uncertainty Shocks: CM vs IM



Two-Channel Spanning (Proposition 2)

Can the responses to tariff uncertainty be mimicked by classical first-order shocks?

- With common shock persistence ρ , responses to tariff uncertainty can be mapped to responses to **productivity shock** and **risk-premium shock** with fixed weights

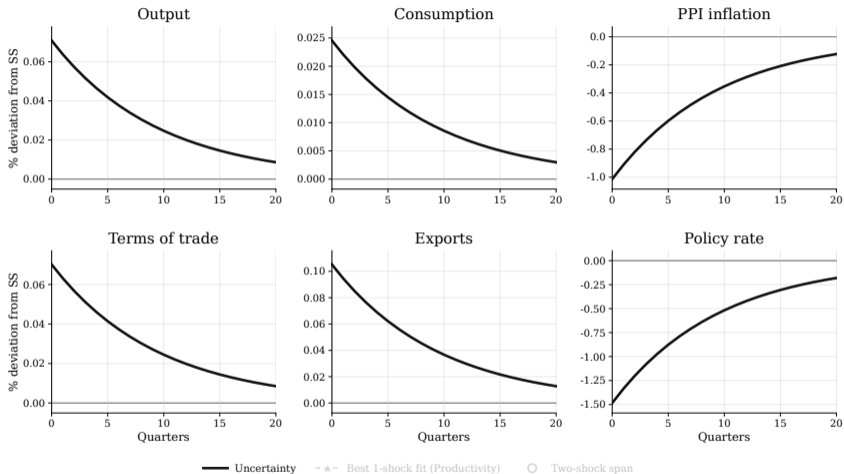
$$\text{IRF}_t^{\text{unc}}(x) = \lambda_A \text{IRF}_t^{\text{Prod}}(x) + \lambda_\psi \text{IRF}_t^\psi(x) + O(4)$$

where

$$\lambda_A = \eta_\sigma \frac{\tilde{\kappa}\chi_q - (1 - \beta\rho)\chi_\pi}{\sigma_A c_a}, \quad c_a \equiv \kappa_{mc}(1 + \varphi)$$
$$\lambda_\psi = \eta_\sigma \frac{-(\phi_y\theta_q + 1 - \rho)\chi_q - (\phi_\pi - \rho)\chi_\pi}{\sigma_\psi}$$

- Baseline: $\lambda_A = 0.316$, $\lambda_\psi = 0.566$

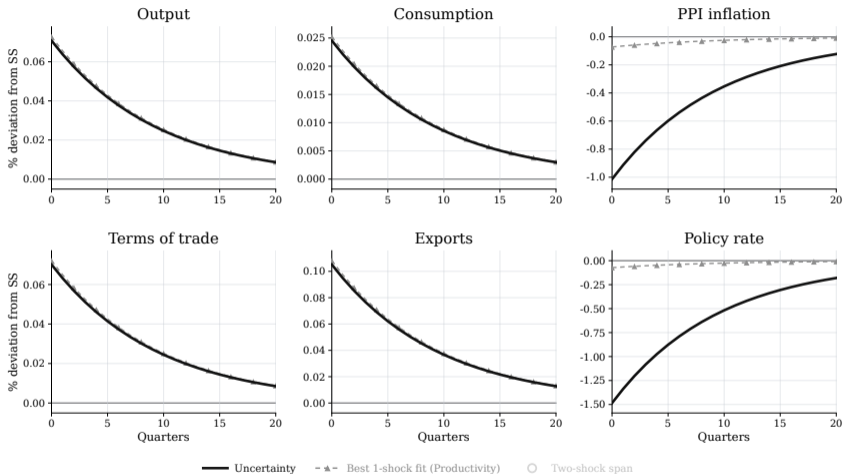
Two Shocks Are Necessary and Sufficient



Spanning

Spanning Weights

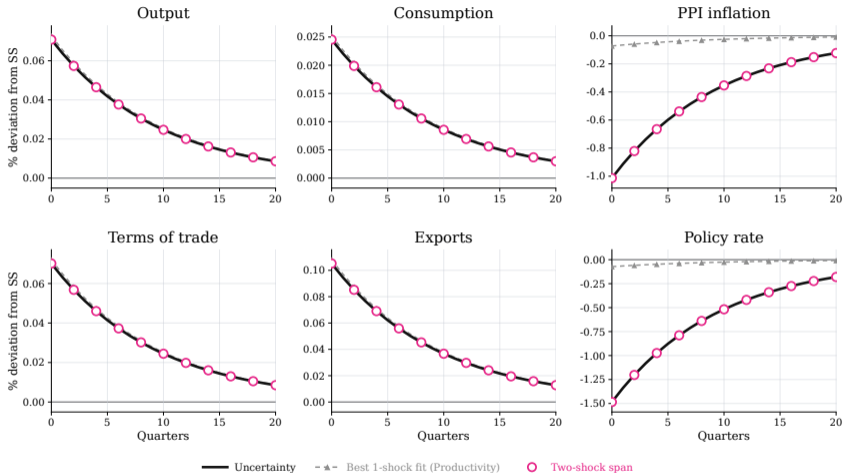
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